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RAPIER - A FORTRAN IV PROGRAM FOR MULTIPLE LINEAR REGRESSION ANALYSIS PROVIDING INTERNALLY EVALUATED REMODELING

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16. Abstract

RAPIER is a very flexible, easy to use, sophisticated multiple linear regression program which computes the variance-covariance matrix of the independent variables, regression coefficients, t-statistics for individual tests, and analysis of variance tables. The major value of the program is its comprehensiveness and options, such as a choice of three strategies for the variance estimate, an analysis of more than one set of response variables for the same independent variables, a backward rejection based on the first response variable, the use of weighted regression, computation of predicted values for any combination of independent variables, and a chi-square test for normality.

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RAPIER - A FORTRAN IV PROGRAM FOR MULTIPLE LINEAR REGRESSION ANALYSIS PROVIDING INTERNALLY EVALUATED REMODELING

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SUMMARY

RAPIER is a digital computer program which can be used with ease to perform extensive regression analyses or a simple least-squares curve fit, and it includes a backward term rejection option. The program is written in FORTRAN IV, version 13, for the IBM 7094/7044 DCS. The major value of the program is its comprehensiveness of calculations and options.

RAPIER computes the variance-covariance matrix of the independent variables, regression coefficients, t-statistics for individual tests, and analysis of variance tables for overall testing of regression. There is a provision for a choice of three strategies for the variance estimate to be used in computing t-statistics.

Also, more than one set of response or dependent variables can by analyzed for the same set of independent variables.

A backward rejection option method based on the first dependent variable may be used. In this case, a critical significance level is supplied as input. The least significant independent variable is deleted and the regression recomputed. This process is repeated until all remaining variables have significantly nonzero coefficients.

The algorithm uses the triangular form of symmetric matrices throughout. It also allows for the use of weighted regression, computation of predicted values at any combination of independent variables, a table of residuals, and a chi-square test for the normality of the distribution of residuals.

INTRODUCTION

RAPIER is an almost entirely new multiple regression computer program. It is the result of 5 years of development in meeting the needs of several statistical investigators posing a variety of problems. The problems included analysis of nuclear reactor components, determining predictive models from corrosion and fracture data of both metals and alloys, investigating the behavior of processing variables in the manufacture of

solar cells, optimizing fuel-cell experiment procedures, and predicting personnel performance from academic histories.

The nucleus of the program is based on a program written by Kunin (ref. 1). However, in its present expanded form, it allows the user to choose from a number of sets of options which include options of input, of methods for calculation, and of output, thereby providing great flexibility.

With the aid of a few control cards, the program can be used readily for a wide range of applications which can vary from a simple least-squares curve-fitting problem to a complete regression analysis. It can provide the variance-covariance matrix of independent variables, regression coefficients, the variance-covariance matrix of the regression coefficients, individual t-statistics with their significance levels, analysis of variance tables for significance of regression, special usage of replicated data to estimate the error due to lack of fit, any one of three pooling procedures which may be used to estimate the error variance, tests for normality of distribution of the residuals, weighted regression, and the use of more than one dependent variable.

The mathematical analysis of the computations and their reliability is aided further by the option of obtaining an eigenvector decomposition of both the variance-covariance matrix and the correlation matrix of the independent variables.

The program also provides an option to perform a backward rejection regression at any given level of significance.

Despite its sophistication, RAPIER is relatively easy to use, but it presupposes that the user has at least a basic knowledge and/or experience in the application of statistical techniques.

To provide a framework for the discussion of the calculations and statistical options available in RAPIER, a brief description of multiple linear regression is presented, with no attempt to make the discussion thorough or rigorous. Notable presentations of applied regression analysis are those by Draper and Smith (ref. 2) and Graybill (ref. 3). Reference 4 by Kendall and Stuart is a useful guide to both applications and theoretical justifications. Rao (ref. 5) presents a more mathematically sophisticated treatment of the subject of linear statistical models.

After discussion of the calculations and options available, the card input necessary is described in detail and illustrated by an example which uses almost all of the options.

SYMBOLS

A	matrix
A'	transpose of A
A^{-1}	inverse of A

B	matrix
b	vector (column)
b_i	true regression coefficient
$\hat{b}_i$	estimated regression coefficient
b_0	constant term
$b_1, \ldots, b_J$	unknown parameters
C	correlation matrix
C_{ij}	elements of C
D	indicator variable, equal to 0 if no b_0 coefficient is estimated and equal to 1 if b_0 is estimated
E(x)	expected value of x (i. e., average of x over all possible values of x)
e	vector of observation errors
$F_{a,b}$	statistic distributed as variance ratio with a and b degrees of freedom
f	expected number of observations in each cell of a partitioned range of studentized residuals
$f_j(z_1, \ldots, z_K)$	term of regression equation
H_0	statistical hypothesis to be tested
H_1	alternate hypothesis to be accepted if H_0 is judged to be false
J	number of coefficients estimated, excluding b_0
K	number of independent variables observed
k	number of segments or cells in range of possible studentized residuals
LOF	lack of fit
M	total number of independent and dependent variables
MS(source)	mean square due to source, where source is REG, RES, etc.
m	moment about origin
N	number of observations
$N(\mu,\sigma^2)$	normal distribution with mean μ and variance σ^2
n_i	number of studentized residuals in i^{th} cell

p_i	probability in i^{th} cell
R	number of sets of replicates
REG	regression
REP	replication
RES	residual
r	number of replicates in set
S	diagonal matrix
S_c	sum of squares correction if $D = 1$, and 0 if $D = 0$
s_j	elements of diagonal matrix
SSQ(source)	sum of squares due to source, where source is REG, RES, etc.
TOT	total
t_n	statistic distributed as Student's t with n degrees of freedom
V(x)	variance of x, expected value of $(x - E(x))^2$
W, X	matrices
x	vector (column)
x(J)	x_j
$\bar{x}_{\cdot j}$	$\frac{1}{N}\sum_{i=1}^{N} x_{ij}$
y	vector (column)
Z_i	studentized residual
$z_1, \ldots, z_K$	variables
μ_x	mean of x defined as E(x)
$\hat{\mu}$	estimate of μ based on observation of random sample
σ_x^2	variance of x defined as V(x)
$\hat{\sigma}^2$	estimate of σ^2 based on observation of random sample
χ_n^2	statistic distributed as chi-square with n degrees of freedom
$\sim$	is distributed as

Superscript:

'	transpose

ESTIMATION OF BASIC LINEAR MODEL

Basic Linear Model

In multiple linear regression, a dependent or response variable Y (such as temperature or pressure) measured on an object or experiment is assumed to be correlated with a function of one or more other variables $(z_1, \ldots, z_K)$ measured on the same object or experiment. This function includes a number of unknown parameters $(b_1, \ldots, b_J)$ and can be represented as

$$y = h(b_1, \ldots, b_J, z_1, \ldots, z_K) + e \tag{1}$$

The only restriction imposed on this function is that it be linear in the parameters

$$y = \sum_{j=1}^{J} b_j f_j(z_1, \ldots, z_K) \tag{2}$$

where $f_j(z_1, \ldots, z_K)$ is a TERM of the regression equation. (A TERM is a quantity which may be a variable or a function of a variable, e.g., T is a TERM and Z, after it is defined as $Z = \log T$, is also a TERM.)

Suppose that there are N observations of the dependent variable. Let the subscript i indicate that the values are associated with the i^{th} observation; in particular, the value of the response variable y_i would depend on the observed values of the variables $(z_{i1}, \ldots, z_{iK})$. Also, let the subscript j denote the j^{th} term in the regression model so that $x_{ij} = f_j(z_{i1}, \ldots, z_{iK})$ describes the transformations of the $(z_{i1}, \ldots, z_{iK})$ to produce the value of x_{ij} for the j^{th} term at the i^{th} observation.

The regression model can now be rewritten as

$$y_i = b_1 x_{i1} + b_2 x_{i2} + \ldots + b_J x_{iJ} + e_i \qquad i = 1, \ldots, N \tag{3}$$

where e_i denotes the difference between the observed value and the expected value of y_i. For the N observations, it is convenient to write this regression model in matrix notation as $y = Xb + e$ where

$$
\left.
\begin{aligned}
y &= \begin{pmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{pmatrix} \\
X &= \begin{pmatrix} x_{11} & \cdots\cdots & x_{1J} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ x_{N1} & & x_{NJ} \end{pmatrix} \\
b &= \begin{pmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b_J \end{pmatrix} \\
e &= \begin{pmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e_N \end{pmatrix}
\end{aligned}
\right\} \tag{4}
$$

More often than not, the analyst feels the model

$$y_i = b_0 + b_1 x_{i1} + \ldots + b_J x_{iJ} + e_i \qquad i = 1, \ldots, N \tag{5}$$

is more appropriate. Let $a_0 = b_0 + b_1 \bar{x}_{.1} + \ldots + b_J \bar{x}_{.J}$. Then, as a result of adding

this equation to, and subtracting it from, equation (5) and rearranging terms

$$y_i = (b_0 + b_1\bar{x}_{.1} + \cdots + b_J\bar{x}_{.J})$$

$$+ b_1(x_{i1} - \bar{x}_{.1}) + \cdots + b_J(x_{iJ} - \bar{x}_{.J}) + e_i \qquad i = 1, \ldots, N \tag{6}$$

If then, a dummy variable x_0 is introduced such that for all values of i, $x_{i0} = 1.0$, equation (6) may be written as

$$y_i = a_0x_{i0} + b_1(x_{i1} - \bar{x}_{.1}) + \cdots + b_J(x_{iJ} - \bar{x}_{.J}) + e_i \qquad i = 1, \ldots, N \tag{6a}$$

Equation (6a) now resembles equation (3) and may be written in matrix notation, similar to equation (4), as $y = Xb + e$ where now

$$\left.\begin{aligned}
y &= \begin{pmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{pmatrix} \\
X &= \begin{pmatrix} 1.0 & x_{11} - \bar{x}_{.1} & \cdots & x_{1J} - \bar{x}_{.J} \\ 1.0 & x_{21} - \bar{x}_{.1} & \cdots & x_{2J} - \bar{x}_{.J} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1.0 & x_{N1} - \bar{x}_{.1} & \cdots & x_{NJ} - \bar{x}_{.J} \end{pmatrix} \\
b &= \begin{pmatrix} a_0 \\ b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_J \end{pmatrix} \\
e &= \begin{pmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e_N \end{pmatrix}
\end{aligned}\right\} \tag{7}$$

Estimating b

Equations (4) and (7) are similar in form and for $N > J$ are an overdetermined set of linear equations. There will be some vector $\hat{b}$ which is a "best" vector to use. If the vector e is composed of random variables e_i such that $E(e_i) = 0$, $V(e_i) = \sigma^2 < +\infty$, and the e_i are uncorrelated, then as is well known, the method of least squares gives the linear minimum variance estimators $\hat{b}$ for b. And $\hat{b}$ is given by

$$\hat{b} = (X'X)^{-1} X'y \tag{8}$$

The matrix $X'X$ divided by $N - 1$ is called the variance-covariance matrix of the independent variables. The variance-covariance matrix of $\hat{b}$ is given by

$$V(\hat{b}) = \sigma^2 (X'X)^{-1} \tag{9}$$

It is important to note that when the form of equation (7) is used, $X'X$ is

$$X'X = \begin{pmatrix} N & 0 & \cdots & 0 \\ 0 & \sum_1^N (x_{i1} - \bar{x}_{.1})^2 & \cdots & \sum_1^N (x_{i1} - \bar{x}_{.1})(x_{iJ} - \bar{x}_{.J}) \\ \vdots & \vdots & & \vdots \\ 0 & \sum_1^N (x_{i1} - \bar{x}_{.1})(x_{iJ} - \bar{x}_{.J}) & \cdots & \sum_1^N (x_{iJ} - \bar{x}_{.J})^2 \end{pmatrix} \tag{10}$$

This is seen to be symmetric and of the form

$$X'X = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$$

Hence,

$$(X'X)^{-1} = \begin{pmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{pmatrix}$$

RAPIER uses this relation to advantage by storing only the upper triangular part of B and computing only the coefficients $b_1, \ldots, b_J$ by matrix manipulations. Then b_0 is given by the simple equation

$$b_0 = \bar{y} - \hat{b}_1\bar{x}_{.1} - \hat{b}_2\bar{x}_{.2} - \ldots - \hat{b}_J\bar{x}_{.J} \tag{11}$$

where $\bar{y} = \sum y_i / N = \hat{a}_0$. It can also be shown that

$$V(\hat{b}_0) = V(\bar{y}) + V(\hat{b}'\bar{x}) = \left[\frac{1}{N} + \bar{x}'(X'X)^{-1}\ \bar{x}\right]\sigma^2$$

$$COV(\hat{b}_0, \hat{b}) = -(X'X)^{-1}\ \bar{x}\sigma^2$$

When there is no b_0 term in the regression model,

$$X'X = \begin{bmatrix} \sum_1^N x_{i1}^2 & \sum_1^N x_{i1}x_{i2} & \cdots & \sum_1^N x_{i1}x_{iJ} \\ \vdots & \vdots & & \vdots \\ \sum_1^N x_{i1}x_{iJ} & \sum_1^N x_{i2}x_{iJ} & \cdots & \sum_1^N x_{iJ}^2 \end{bmatrix} \tag{12}$$

Comparing this to equation (10) shows this form of X'X to be similar to the lower right submatrix in equation (10). This similarity is used to simplify notation by assuming that X'X represents either the form of equation (12) or the lower right portion of equation (10) and considering the calculation of b_0 as a special case. Thus, further reference to b implies

$$b = \begin{pmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_J \end{pmatrix}$$

Correlation Matrix

Another matrix of interest both computationally and statistically is the correlation matrix C. The elements of C, which are denoted C_{ij}, are the sample correlation coefficients between the terms X_i and X_j. These are

$$C_{ij} = \sum_{l=1}^{N} \frac{(x_{li} - \bar{x}_{.i})(x_{lj} - \bar{x}_{.j})}{\sqrt{\sum_{l=1}^{N} \left(x_{li} - \bar{x}_{.i}\right)^2 \sum_{l=1}^{N} \left(x_{lj} - \bar{x}_{.j}\right)^2}} \tag{13}$$

and all these numbers are between 1.0 and -1.0.

The calculation of C can be expressed in matrix notation conveniently by defining a diagonal matrix $S = \mathrm{diag}(s_1, s_2, \ldots, s_J)$ with elements

$$s_j = \frac{1.0}{\sqrt{(X'X)_{jj}}} \qquad j = 1, \ldots, J \tag{14}$$

Then

$$C = S(X'X)S \tag{15}$$

and

$$(X'X)^{-1} = S\left[S^{-1}(X'X)^{-1}S^{-1}\right]S = SC^{-1}S \tag{16}$$

The algorithm of RAPIER performs the following operations: (1) constructs the X'X matrix, (2) computes C, (3) inverts C, (4) computes $(X'X)^{-1}$ from C^{-1} by equation (16), and (5) computes the b estimates. Because C is a normalized matrix, the

inversion of C is likely to be more accurate than direct inversion of X'X. Examination of the structure of X'X and/or C is of assistance in evaluating the possible numerical problems.

It may also be that the independent variables are random variables. Then X'X divided by N - 1 represents the variance-covariance matrix and C the sample correlation matrix. If the independent variables are considered to be from a multivariate distribution, it is useful in some cases to consider the eigenvalues and eigenvectors of X'X and/or C.

For these reasons, RAPIER includes options to compute and print these quantities. As a partial check on the accuracy of the inversion process, it is also possible to have $C \cdot C^{-1}$ computed and printed. This should be the identity matrix.

Estimating σ^2

For any regression model $y = Xb + e$, there are possibly two methods of estimating σ^2. First, if the assumed regression model is, in reality, the true model, it is well known that an unbiased estimator is given by

$$\sigma^2_{RES(J)} = \frac{y'y - \hat{b}'x'y}{N - J - D}$$

$$= \frac{SSQ(RES)}{N - J - D}$$

$$= MS(RES(J)) \tag{17}$$

Second, where there are replicated data points, another estimator of σ^2, depending only on $V(e_i) = \sigma^2$ for all i and not on the correctness of the assumed model, is the pooled mean squares computed from the replicated data points.

Assume the observations are grouped into replicate sets in sequence. Let R be the number of sets of replicates and r_i be the number of replicates in the i^{th} replicate set. Let

$$SSQ(i) = \sum_{n=r^*+1}^{r^*+r_i} \left(y_n - \bar{y}_i\right)^2 \tag{18}$$

where

$$r^* = \sum_{j=1}^{i-1} r_j$$

It is assumed y_n is from the i^{th} replicate set and $\bar{y}_i$ is calculated only from those y_n in the i^{th} replicate set. Then define the pooled sum of squares due to replication as $SSQ(REP) = \sum_{i=1}^{R} SSQ(i)$ and the pooled degrees of freedom as $NPDEG = \sum_{i=1}^{R} (r_i - 1)$. The second estimator of σ^2 becomes

$$\sigma^2_{REP} = \frac{SSQ(REP)}{NPDEG}$$

$$= MS(REP) \tag{19}$$

It can be shown (ref. 2) that the sums of squares due to residuals can be partitioned into a component due to replication and a component due to lack of fit; that is,

$$SSQ(RES) = SSQ(LOF) + SSQ(REP) \tag{20}$$

This partitioning is used later to determine the estimate of σ^2 to use in tests of hypotheses.

HYPOTHESIS TESTING

Test NE - Normality of e

As stated before, the only assumption necessary for $\hat{b}$ to be a linear minimum variance estimator is that $E(e_i) = 0.0$, $V(e_i) = \sigma^2 < +\infty$, and e_i be uncorrelated. If it can further be assumed that $e_i \sim N(0, \sigma^2)$, a number of standard tests become available. RAPIER computes a chi-square statistic and the sample skewness and kurtosis for testing this hypothesis.

Under the hypothesis $e_i \sim N(0, \sigma^2)$, the studentized resiudals defined by

$$Z_i = \frac{y_i - \hat{y}_i}{\hat{\sigma}} = \frac{e_i}{\hat{\sigma}}$$

will be distributed as Student's t with the degrees of freedom associated with the estimate $\hat{\sigma}$. If the degree of freedom is 30 or more, the t distribution is very close to the normal.

The range of possible studentized residuals is $(-\infty, +\infty)$ and may be divided into k segments or cells each with probability p_i, so that each segment will have Np_i as the expected number of observations falling into it. Let n_i denote the number of studentized residuals in the i^{th} cell. Then a chi-square goodness-of-fit statistic may be calculated as

$$\chi^2_{(k-1)} = \sum_{i=1}^{k} \frac{\left(n_i - Np_i\right)^2}{Np_i}$$

RAPIER computes this statistic by using an even number of cells greater than or equal to four and less than or equal to 20, such that the expected numbers of observations per cell is five or more. The bounding values for the i^{th} cell are Z_{i-1}, Z_i where $F(Z_i) = (i \cdot k)/N$ and $F(Z)$ is the cumulative normal distribution function. Then each cell has the same expected number of observations, say $f = N/k$. Then

$$\chi^2_{(k-1)} = \sum_{i=1}^{k} \frac{\left(n_i - f\right)^2}{f} = \frac{k}{N} \sum_{i=1}^{k} n_i^2 - N$$

This statistic is not computed for less than 30 degrees of freedom for the estimate $\hat{\sigma}^2$.

Two other statistics which may be used to test the normality of an empirical distribution are skewness and kurtosis. Define the moments about the origin as

$$m_2 = \frac{1}{N} \sum Z_i^2$$

$$m_3 = \frac{1}{N} \sum Z_i^3$$

$$m_4 = \frac{1}{N} \sum Z_i^4$$

where Z_i is the i^{th} studentized deviate. Then skewness is $RELSKW = m_3^2/m_2^3$, which should be nearly zero. Kurtosis is $RELKUR = m_4/m_2^2$, which should be nearly 3. Probability points for these are tabulated in reference 6.

If these statistics indicate nonnormality, there are three possible courses of action. First, perhaps a transformation of the response variable or the independent variables can be found which will bring the distribution of residuals closer to normal. RAPIER makes this task quite easy. Second, a different candidate model might be used (see ref. 2, "Analysis of Residuals"). As the last choice, it is possible to do nothing and simply rely on the robustness of the tests involved. See reference 4 for definition and discussion of robustness.

Also note that the individual observations may be weighted to perform a weighted regression analysis. RAPIER permits the use of weights (ref. 2). In this case, the X'X and X'y matrices take the form

$$\left.\begin{aligned}
X'X &= \begin{pmatrix} \sum_{i=1}^{N}\left[\left(x_{i1} - \bar{x}_{.1}\right)^2 w_i\right] & \cdots & \sum_{i=1}^{N}\left[w_i(x_{i1} - \bar{x}_{.1})(x_{iJ} - \bar{x}_{.J})\right] \\ \vdots & & \vdots \\ \sum_{i=1}^{N}\left[(x_{iJ} - \bar{x}_{.J})(x_{i1} - \bar{x}_{.1})w_i\right] & \cdots & \sum_{i=1}^{N}\left[\left(x_{iJ} - \bar{x}_{.J}\right)^2 w_i\right] \end{pmatrix} \\
X'y &= \begin{pmatrix} \sum_{i=1}^{N} x_{i1}y_iw_i \\ \vdots \\ \sum_{i=1}^{N} x_{iJ}y_iw_i \end{pmatrix}
\end{aligned}\right\} \tag{21}$$

Analysis of Variance Table

For most hypothesis testing of the regression model, it is convenient to summarize the available information in an Analysis of Variance (ANOVA) table, as follows:

Source	Sums of squares	Degrees of freedom	Mean squares
Regression (REG)	$SSQ(REG) = \hat{b}'X'y - S_c$[a]	J	MS(REG) = SSQ(REG)/J
Residual (RES)	$SSQ(RES) = y'y - \hat{b}'X'y$	N - J - D[b]	MS(RES) = SSQ(RES)/(N - J - D)
Total	$SSQ(TOT) = y'y - S_c$.	N - D	

[a] $S_c = \begin{cases} 0 \text{ if no } b_0 \text{ coefficient is estimated.} \\ N\bar{y}^2 \text{ if a } b_0 \text{ coefficient is estimated.} \end{cases}$

[b] $D = \begin{cases} 0 \text{ if no } b_0 \text{ coefficient is estimated.} \\ 1 \text{ if } b_0 \text{ is estimated.} \end{cases}$

If there are replicated data points, another ANOVA table can be constructed to show the separation of the residual sums of squares into components from lack of fit and replication, as in the following table:

Source	Sums of squares	Degrees of freedom	Mean squares
Lack of fit (LOF)	SSQ(LOF) = SSQ(RES) - SSQ(REP)	N - J - D - NPDEG	MS(LOF) = SSQ(LOF)/(N - J - D - NPDEG)
Replication (REP)	SSQ(REP)	NPDEG	MS(REP) = SSQ(REP)/NPDEG
Residual (RES)	$y'y - \hat{b}'X'y$	N - J - D	

Choice of Estimator for σ^2

As mentioned previously, there are two possible methods of estimating σ^2 depending on whether there are replicated data points. This is true for any given model equation. When the backward rejection option of RAPIER is used, there is no longer one hypothetical model but a series of models. Thus, there is the choice of estimator for σ^2 to be made after each rejection of a term in the previous model.

As an example, consider the model

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + e \tag{22}$$

with replicated data points. The first step is to estimate b_0, b_1, b_2, and b_3. There will then be the estimators $\hat{\sigma}^2_{RES(J)}$ and $\hat{\sigma}^2_{REP}$. If the model in equation (22) has not left out any significant terms, both estimators are valid.

The ratio $F = MS(LOF)/MS(REP)$ can be used to test the hypothesis that there is no lack of fit, where $F \sim F_{a,b}$ with $a = N - J - D - NPDEG$ and $b = NPDEG$ degrees of freedom. If the test accepts the hypothesis of no lack of fit, MS(RES) is a pooled estimate of σ^2 with more degrees of freedom and will usually make tests using $\hat{\sigma}^2_{RES(J)}$ more sensitive than those using $\hat{\sigma}^2_{REP}$. But there is the possibility that the hypothesis was accepted as a result of random fluctuation when there really is some lack of fit; that is, there is the possibility that $\hat{\sigma}^2_{RES(J)}$ is a biased estimator. If lack of fit is not concluded to be significant, the decision to pool or not is usually made on the basis of the number of degrees of freedom for replication. If this is "large" (no definition of large is given herein), $\hat{\sigma}^2_{REP}$ is used. If "small," the pooled estimate $\hat{\sigma}^2_{RES(J)}$ is used.

In testing equation (22), should it be decided that b_3 is not significantly different from zero (see section Test TT - t-Tests), the coefficients of the following model would be estimated:

$$y = b_0 + b_1x_1 + b_2x_2 + e$$

From this model there is an estimate $\hat{\sigma}^2_{RES(J-1)}$. This estimate could also be biased since b_3 may be small but nonzero and the decision of $b_3 = 0$ may have been due to random fluctuation.

At the first step, the lack of fit can be considered a random sample of an infinite possibility of biases. But the biases due to pooling mean squares after rejecting terms can be considered to be systematic biasing and hence less desirable.

RAPIER provides three strategies of pooling estimates for use in the decision procedure:

(1) Never pool. This is appropriate only when there are replicated data points. The estimator used in all t-tests is $\hat{\sigma}^2_{REP}$.

(2) Always pool initial residual. This will always pool the lack of fit and replication from the first model only. Additional mean squares due to rejected terms will be ignored.

(3) Always pool. This strategy will always use $\hat{\sigma}^2_{RES(J-i)}$ for the model with i rejected terms.

A rule for pooling lack of fit and replication mean squares is discussed by Draper and Smith (ref. 2). Related work as applied to factorial designs is presented by Holms (ref. 7) and Bozivich, Bancroft, and Hartley (ref. 8).

Test OR - Overall Regression

One of the first tests usually applied to a regression model is the test of the overall significance of the model. In the notation of hypothesis testing this is stated H_0: $b = 0$; H_1: $b \neq 0$ where

$$b = \begin{pmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_J \end{pmatrix}$$

The statistic for this test is $F = MS(REG)/\hat{\sigma}^2$. Then $F \sim F_{a,b}$ with $a = J - D$, and b equals the degrees of freedom associated with $\hat{\sigma}^2$.

Another useful statistic for judging the significance of overall regression is $R^2 = SSQ(REG)/SSQ(TOT)$. The sampling distribution of R does not lend itself to very simple tests except in the case of H_0: $R = 0$. The main value of R^2 is that it must be a number in the range 0 to 1 and 100 R^2 is a measure of the percentage of variation in the y values that is accounted for by the regression model.

Test SF - Sequential F-Test

There is often reason to consider a partitioned form of $b' = \left\{w_1', w_2'\right\}$ for testing the hypotheses

$$H_0: \; w_2' = (b_{p+1}, \; . \; . \; ., \; b_J) = 0$$

$$H_1: \; w_2' \neq 0$$

Partition the matrix X corresponding to the partitioning of b and denote it as $X = (W_1, W_2)$ where W_1 is $N \times p$ and W_2 is $N \times (J - p)$. Then the test statistic is $F = (SSQ(REG)_{(p)}/p)\hat{\sigma}_{RES(J)}$, where $SSQ(REG)_{(p)} = \hat{w}_1' W_1' y$ and $\hat{w}_1 = (W_1' W_1)^{-1} W_1' y$. Then $F \sim F_{a,b}$ with $a = p$ and $b = N - p - D$. Sometimes this test is performed with $p = 1$, $p = 2, \; . \; . \; ., \; p = J$. This is then referred to (ref. 2) as the sequential F-test. RAPIER computes regressions for $p = 1, \; . \; . \; ., \; p = J$ upon request.

Test TT - t-Tests

In many cases, the regression model contains terms whose estimated coefficients are "small." This may be an indication that the term does not have a real effect on the dependent variable and that the coefficient is nonzero due to random sampling variation. If this is true, it is desirable to delete the term from the regression model. A test statistic for deciding this is

$$t = \frac{\hat{b}_i}{\hat{\sigma}^2 (X'X)^{-1}_{ii}} \tag{23}$$

where $(X'X)^{-1}_{ii}$ denotes the i^{th} diagonal element of the $(X'X)^{-1}$ matrix. The statistic $t \sim t_{N-J-D}$. An equivalent test statistic is

$$F = t^2 = \frac{\hat{b}_i^2}{\hat{\sigma}^2 (X'X)^{-1}_{ii}} \tag{24}$$

where $F \sim F_{1,N-J-D}$. This is often referred to (ref. 2) as the partial F-test. The quantity $\hat{b}_i \Big/ \left[(X'X)^{-1}_{ii}\right]$ is called the sum of squares due to b_i, if x_i were last to enter the equation. RAPIER computes and prints the t-statistics and the probability associated with the interval $(-t, t)$.

This particular test is the basis for the rejection option of RAPIER. The analyst has chosen which $\hat{\sigma}^2$ estimator to use by the choice of strategy. Then the analyst may choose a significance level which all coefficients must meet. For example, suppose a

significance level of 0.900 is chosen. The t-statistic is then computed for each coefficient, and the coefficient with minimum $|t|$ is identified. If $\min|t| > t_{N-J-D,\,0.950}$, all terms are concluded to be significant at the 0.900 (or 90.0 percent) level of significance. If $\min|t| < t_{N-J-D,\,0.950}$, the term corresponding to the minimum $|t|$ is dropped from the hypothetical model, and the regression is recomputed. This process is repeated until all remaining coefficients are significant at the specified level of probability.

PREDICTING VALUES FROM ESTIMATED REGRESSION EQUATION

Regression equations are often used to predict an estimated response at some condition of the independent variables. Useful estimates of parameters to know are the variance of the regression equation and the variance of a single further observation at the desired combination of the independent variables.

Let $x' = (x_1, \ldots, x_J)$ denote the vector of independent variables at which a prediction is desired. Let $x^* = x - \bar{x}$. Let $\hat{\sigma}^2_{\mu \cdot x}$ denote the estimated variance of the regression equation at x. Let $\hat{\sigma}^2_{y \cdot x}$ denote the estimated variance of a single further observation at x. Then,

$$\hat{\sigma}^2_{\mu \cdot x} = \hat{\sigma}^2 \left[\frac{D}{N} + x^{*\prime}(X'X)^{-1}\, x^*\right] \tag{25}$$

$$\hat{\sigma}^2_{y \cdot x} = \hat{\sigma}^2 \left[1.0 + \frac{D}{N} + x^{*\prime}(X'X)^{-1}\, x^*\right] \tag{26}$$

where, as before, $D = 1$ if a b_0 coefficient is estimated and $D = 0$ if a b_0 coefficient is not estimated. The quantity $s = \hat{\sigma}_{RES(J)}$ is called the standard error of estimate and often is used as a simple approximation to $\hat{\sigma}_{y \cdot x}$. This approximation is close if N is very large and $x = \bar{x}$, in which case,

$$\hat{\sigma}_{y \cdot x} = s\left(1.0 + \frac{D}{N}\right) \approx s$$

When $x \neq \bar{x}$, this may be a poor approximation. RAPIER accepts input vectors x and computes $\hat{y} = \hat{b}_0 + \hat{b}_1 x_1 + \ldots + \hat{b}_J x_J$, as well as $\hat{\sigma}^2_{\mu \cdot x}$, $\hat{\sigma}_{\mu \cdot x}$, $\hat{\sigma}^2_{y \cdot x}$, $\hat{\sigma}_{y \cdot x}$, and the standard error of estimate.

USER'S GUIDE TO INPUT

Sample Regression Problem

Let

x_1 temperature

x_2 time

x_3 pressure

y_1 output, lb

y_2 cost of operation

The data are coded into standardized units, as is often done in experimental design analysis. The y_1 variable is assumed to be of primary interest in this problem.

Table I contains the x and y data. Table II contains a summary of the type of in-

TABLE I. - x AND y DATA

Group	x_1	x_2	x_3	y_1	y_2	Group	x_1	x_2	x_3	y_1	y_2
1	-1	-1	-1	9.17	38.5	8	0	0	0	9.61	49.1
2	1	-1	-1	12.76	43.1					10.01	49.3
	1	-1	-1	12.97	44.0					10.12	50.1
							↓	↓	↓	9.95	51.8
3	-1	1	-1	9.11	58.3	9	-2	0	0	11.78	50.1
	-1	1	-1	8.96	58.7						
4	1	1	-1	17.03	63.2	10	2	0	0	23.83	57.6
							2	0	0	22.90	58.1
5	-1	-1	1	9.05	38.7	11	0	-2	0	7.99	28.6
	-1	-1	1	8.86	39.6						
6	1	-1	1	12.60	44.1	12	0	2	0	12.11	71.0
	1	-1	1	13.21	43.8		0	2	0	11.70	70.2
7	1	1	1	17.20	62.1	13	0	0	-2	10.11	48.7
	1	1	1	17.04	62.8		0	0	-2	10.01	49.9
						14	0	0	2	10.02	50.8

TABLE II. - FUNCTIONS OF INPUT CARDS

Type of input	Function
1	Identification
2	Definition of problem size
3	Definition of problem logic
4	Terms, transformations, and constants
5	Control rejection option
6	Provide replication information
7	Data input unit and format
8	Data
9	Prediction information

SAMPLE INPUT

FORTRAN STATEMENT

```
Type Line
 1     1   15 SAMPLE RAPIER PROBLEM
 1     2      MODEL EQUATION      I=1,2
 1     3
 1     4         Y(I) = BO    +     B1*X1      +B2* X2      +B3*X3
 1     5                         + B12* X1* X2  +B13* X1*X3  +B23* X2*X3
 1     6                         + B11*X1**2    +B22*X2**2   +B33*X3**2   + ERR
 1     7
 1     8         X1 =TEMP                 **
 1     9         X2 =TIME                 ** DATA CODED TO STANDARDIZED UNITS
 1    10         X3 =PRESS                **
 1    11
 1    12         Y1= POUNDS OUTPUT
 1    13         Y2= COST OF OPERATION
 1    14
 1    15         FICTITIOUS DATA REPRESENTING AN INCOMPLETE FACTORIAL
 1    16         DESIGN WITH SOME REPLICATION
 2    17       3  2   9   25
 3    18   TTTTTTTTF1
 4    19     14   0
 4    20   111213141516171819 2021
 4    21   010000110100006102000012020000620300001303000063610261170202621863026319610262 14
 4    22   610263156202631604000020050000 21
 5    23   T950
 6    24   T
 6    25     14   1   2   2   1   2   2   2   4   1   2   1   2   2   1
```

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SAMPLE INPUT (Concluded)

FORTRAN STATEMENT

```
 7    26   05   (5F6.0)
 8    27       -1     -1     -1   9.17  38.5
 8    28        1     -1     -1  12.76  43.1
 8    29        1     -1     -1  12.97  44.0
 8    30       -1      1     -1   9.11  58.3
 8    31       -1      1     -1   8.96  58.7
                 .
                 .
                 .
 8    48        0      0     -2  10.11  48.7
 8    49        0      0     -2  10.01  49.9
 8    50        0      0      2  10.02  50.8
 9    51   T
 9    52       1
 9    53       -1      1      1
```

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Figure 1. - Sample input form.

put cards and their basic functions. Figure 1 shows a sample set of data for a complete regression as it would be written on a FORTRAN coding sheet.

Use the model equations

$$y_i = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + e \qquad i = 1, 2 \tag{27}$$

Test whether the interaction terms as a group are significant, given that the linear terms are in the model. Predict the response at the point $(x_1, x_2, x_3) = (-1, +1, +1)$.

Types of Input Cards

Nine types of data card may be used to define a regression analysis for RAPIER. A summary of the types and their functions is presented in table II.

Type 1. - In type 1 input, as many as 100 cards with Hollerith information may be read to identify and describe the problem. At least one card is read. The first two columns of the first card used to specify the additional number of identification cards to be read, and columns 3 to 80 are used for Hollerith information. Each following card uses columns 1 to 78. (See lines 1 to 16 in fig. 1.)

Type 2. - In type 2 input, one card with three four-column fields followed by a five-column field specifies

(1) Number of independent variables to read

(2) Number of dependent variables to read

(3) Number of terms in the model equation (not counting b_0)

(4) Number of observations

(See line 17 in fig. 1.)

Type 3. - In type 3 input, one card with 10 one-column fields specifies

(1) The b_0 term in the model equation (T or F)

(2) Computation of t-statistics and their confidence levels (T or F)

(3) Weighting factor either of 1.0 (T) or supplied with each data point (F)

(4) Computation of residuals and chi-square test (T or F)

(5) Computation of eigenvalues and eigenvectors of correlation matrix (T or F)

(6) Computation of eigenvalues and eigenvectors of X'X (T or F)

(7) Computation of product of correlation matrix and its inverse (T or F)

(8) Use of bordering inversion technique for computation of sequential regression (T or F); see Test SF - Sequential F-Test

(9) Use of an economy version of output which does not print the matrices $X'X$, $(X'X)^{-1}$, $x'y$, C, or C^{-1} (T or F) (If item 7 of this set is set T, then $C \cdot C^{-1}$ is printed.)

(10) The pooling strategy:

(1) Never pool. Always use replication error. (If there is no replication, the program sets this to 3.)

(2) Pool initial residual.

(3) Pool all residuals.

(See line 18 in fig. 1.)

Type 4, type 4A, type 4B, and type 4C. - Type 4 card has two four-column fields specifying

(1) Number of transformations

(2) Number of constants

(See line 19 of fig. 1.)

If the number of transformations is zero, and therefore the number of constants is zero, the type 4A, type 4B, and type 4C cards are not expected by the program. In this case the program assumes the independent and the dependent variables are arranged on the input cards as

$$x_1, x_2, \ldots, x_J, \; y_1, \ldots, y_{NODEP}$$

where NODEP is the number of dependent variables.

If a weighting factor other than 1.0 is to be used (i.e., if item 3 of the type 3 card contains an F), the value of the weighting factor for each observation must appear as the last item in the list, so that in this case the data for one observation is entered on the cards as

$$x_1, x_2, \ldots, x_J, \; y_1, \ldots, y_{NODEP}, WT$$

For each observation, RAPIER reads a total of M numbers, where M is the sum of the number of independent and dependent variables. These numbers are stored consecutively in an array called VAR, beginning with location 01 and ending with location M. If the weighting factor is not identically 1.0, then M + 1 numbers are read, but the last number, being the weighting factor, is treated and stored separately. The data in VAR are used with appropriate weighting factors to cumulatively create $X'X$ and $X'y$ as shown in equations (21).

When a more complex model is desired, information must be supplied instructing the program as to (1) where to find the values for the TERMS of the equation, (2) how to

create the TERMS from the variables and constants, and (3) what the values are for the constant terms. This can be achieved easily by use of the type 4A(TERMS), type 4B(TRANSFORMATIONS), and type 4C(CONSTANTS) control cards (see p. 29). These three types and their functions can best be described by considering the sample model given by equation (27) as an example which illustrates their application.

An array called CON has a twofold purpose. First, if the number of constants designated in the second field of the type 4 card is nonzero, that number of constants will be read from the type 4C card and stored consecutively in this array beginning with location 01. If the number of constants is zero, the type 4C card is not expected by the program. Second, all the intermediate and final results of transformations are also stored in the CON array as the program obeys the instructions of the type 4B cards. The type 4A card must identify the relative location in the CON array where the value for each TERM is to be found for constructing the X'X and X'y matrices.

The VAR and CON arrays for this example are illustrated in figure 2. Five numbers are read for each observation: x_1, x_2, x_3, y_1, and y_2. These numbers automatically enter the VAR array beginning with location 01. Using transformation codes packed in fields of eight columns each on the type 4B cards, the program stores the result of each transformation into the appropriate relative location in the CON array as designated by

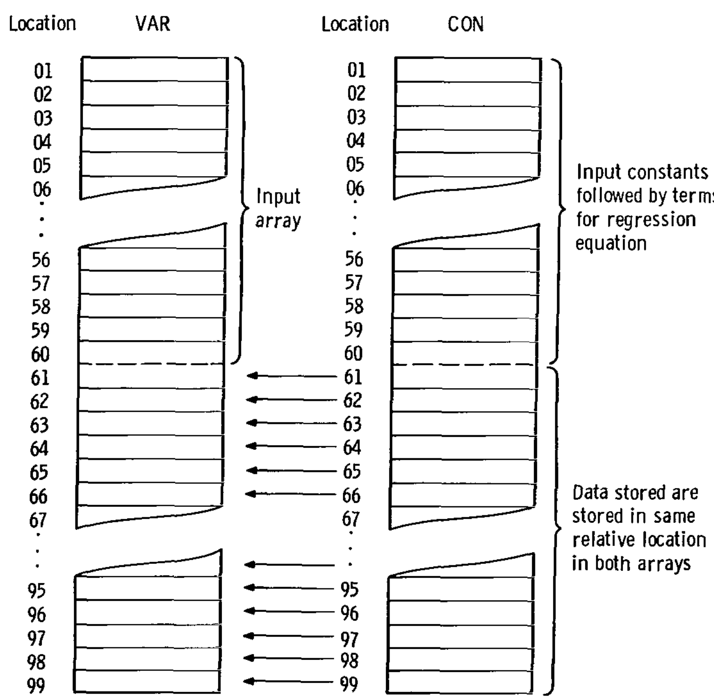


Figure 2. - Map of VAR and CON arrays. Data transferred into any location of CON array beyond location 60 are immediately duplicated in same relative location in VAR array.

the last two digits of the field. Each transformation code is made up of four subfields of two card columns each, with the following interpretation:

Subfield	Interpretation	
1	Operand VI	Relative location in VAR
2	Operation OP	Arithmetic operation
3	Operator CI	Relative location in CON
4	Result CS	Relative location in CON

Thus, subfield 1 always references the VAR array and subfields 3 and 4 reference the CON array. The result of every transformation is a term which is stored in the designated location of the CON array, with the added feature that if the term is stored in relative location 61 or beyond, it is also stored in the VAR array. This is illustrated by the arrows in figure 2. This feature allows successive transformations to be performed more easily. The OP (operation codes) are tabulated in table III.

TABLE III. - OPERATIONS[a] AND CODE NUMBERS

Operation code (OP)	Resulting operation	Operation code (OP)	Resulting operation
00	No operation	16	1.0/SQRT(VAR)
01	VAR + CONST	17	CONST**VAR
02	VAR*CONST	18	10.0**VAR
03	CONST/VAR	19	SINH(VAR)
04	EXP(VAR)	20	COSH(VAR)
05	VAR**CONST	21	(1.0-COS(VAR))/2.0
06	ALOG(VAR)	22	ATAN(VAR)
07	ALOG10(VAR)	23	ATAN2(VAR/CONST)
08	SIN(VAR)	24	VAR**2
09	COS(VAR)	25	VAR**3
10	SIN(π*CONST*VAR)	26	ARCSIN(SQRT(VAR))
11	COS(π*CONST*VAR)	27	2.0*π*VAR
12	1.0/VAR	28	1.0/(2.0*π*VAR)
13	EXP(CONST/VAR)	29	ERF(VAR)
14	EXP(CONST/VAR**2)	30	GAMMA(VAR)
15	SQRT(VAR)		

[a]All function names and operations are consistent with FORTRAN IV mathematical subroutines.

TABLE IV. - SEQUENCE OF TRANSFORMATIONS

Transformation number	VI	OP	CI	CS	Interpretation
1	01	00	00	11	$x_1 \rightarrow$ CON(11)
2	01	00	00	61	$x_1 \rightarrow$ VAR(61), CON(61)
3	02	00	00	12	$x_2 \rightarrow$ CON(12)
4	02	00	00	62	$x_2 \rightarrow$ VAR(62), CON(62)
5	03	00	00	13	$x_3 \rightarrow$ CON(13)
6	03	00	00	63	$x_3 \rightarrow$ VAR(63), CON(63)
7	61	02	61	17	$x_1^2 \rightarrow$ CON(17)
8	62	02	62	18	$x_2^2 \rightarrow$ CON(18)
9	63	02	63	19	$x_3^2 \rightarrow$ CON(19)
10	61	02	62	14	$x_1x_2 \rightarrow$ CON(14)
11	61	02	63	15	$x_1x_3 \rightarrow$ CON(15)
12	62	02	63	16	$x_2x_3 \rightarrow$ CON(16)
13	04	00	00	20	$y_1 \rightarrow$ CON(20)
14	05	00	00	21	$y_2 \rightarrow$ CON(21)

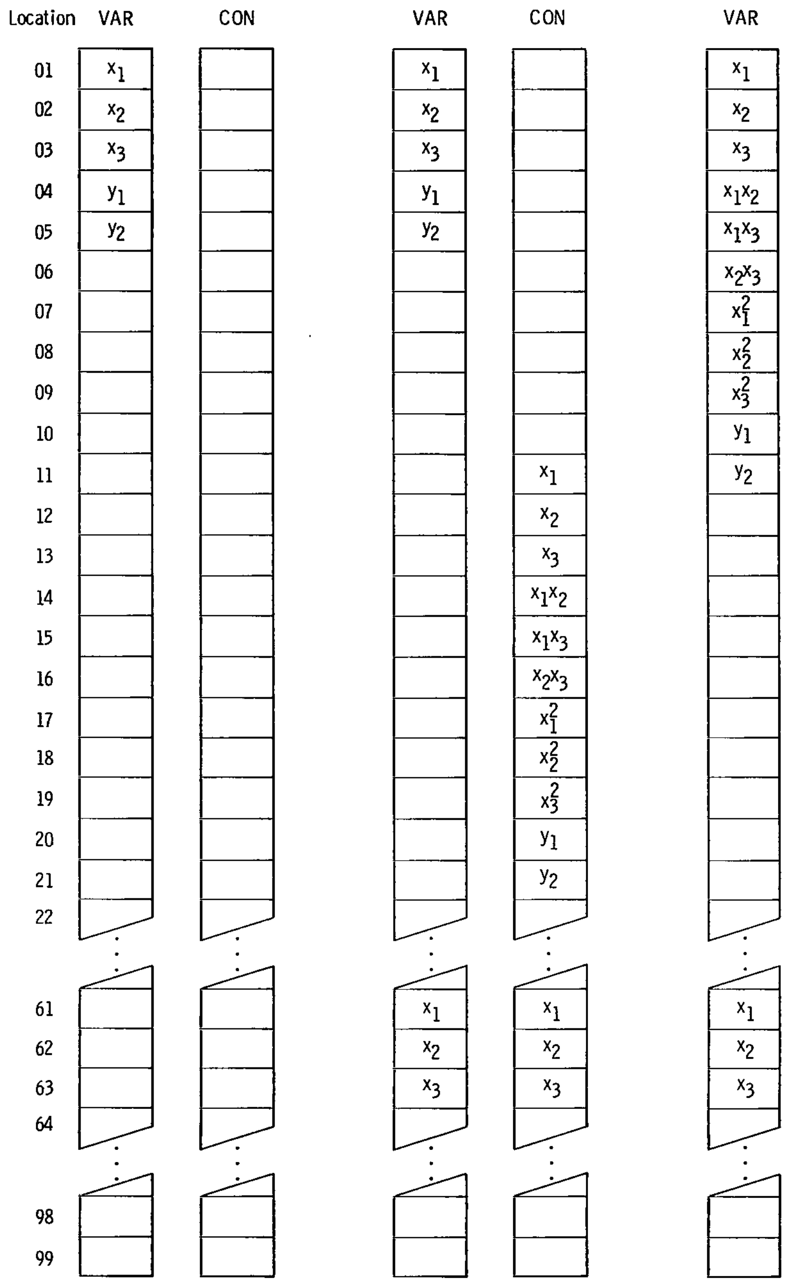
Table IV shows the sequence of transformations used to construct the terms of the example in equation (27). (See lines 21 and 22 of fig. 1.)

The arrays VAR and CON are shown in figure 3 both before and after one set of transformations performed on an observation. Note that CON now contains x_1, x_2, x_3, x_1x_2, x_1x_3, x_2x_3, x_1^2, x_2^2, x_3^2, y_1, y_2 along with unused locations. It may be that not all of these quantities are needed to express the model equation. The type 4A(TERMS) card must be used to supply the locations of CON which contain the terms of the model equation. (Note that this allows the user to somewhat arbitrarily assign terms to locations in CON.) The terms identifying the independent variables must be first, and the terms identifying the dependent variables last, just as in the assumed convention when no transformations are performed. Thus, in this case the terms needed are, according to equation (27),

x_1	x_2	x_3	x_1x_2	x_1x_3	x_2x_3	x_1^2	x_2^2	x_3^2	y_1	y_2
11	12	13	14	15	16	17	18	19	20	21

(See line 20 of fig. 1.)

After the set of transformations has been performed on an observation, the contents of the relative locations of the CON array specified on the terms card are transferred



(a) Before transformations.

(b) After transformations.

(c) After terms selection.

Figure 3. - Arrays VAR and CON before and after transformations and terms selection, for the first example.

back to VAR in consecutive locations beginning with location 01. Thus, for this example, after selection of proper terms, the VAR array contains

$$x_1, x_2, x_3, x_1x_2, x_1x_3, x_2x_3, x_1^2, x_2^2, x_3^2, y_1, y_2$$

in consecutive locations as required by equation (27) and the convention on independent and dependent variables. The b_0 term is accounted for by setting the proper item of type 3 input to T.

There are three important facts to note concerning types 4A, 4B, and 4C. First, the transformation with OP = 00 is an identity transformation which simply transfers data from VAR to CON. If transformations are desired at all, a minimum requirement is that all variables at least be moved to CON so that the selection of terms will have a number to move back to VAR. Second, constants used in the transformations are stored in CON in locations beginning with 01 in sequence. Suppose there are NC constants initially supplied. Then if a transformation has any of the locations 01 through NC referenced in subfield 4, a term will replace the constant. Third, the use of TRANSFORMATIONS and TERMS overrides the convention that independent variables must precede dependent variables on the input data cards. The convention holds true for the terms card data in this case.

The sequence of input and formats is as follows:

(1) Type 4A(TERMS): One or more cards, as necessary, with two-column fields denoting the relative locations of the CON array containing the final terms to be used in regression model (Up to 60 independent and nine dependent terms may be supplied. See line 20 of fig. 1.)

(2) Type 4B(TRANSFORMATIONS): As many cards as necessary, with 10 transformations per card, each transformation being composed of four two-column subfields (A maximum of 100 transformations may be performed. See lines 21 and 22 of fig. 1.)

(3) Type 4C(CONSTANTS): As many cards as necessary, containing the required number of constants in (5E15.7) format. As many as 60 constants may be supplied.

A second example is shown to illustrate the flexibility afforded by the TERMS, TRANSFORMATIONS, and CONSTANTS. Consider a model with two independent and two dependent variables with four terms given by

$$b_0 + b_1x_1 + b_2x_2 + b_3x_1x_2 + b_4x_1^3 = y_1 = \frac{k}{y_2}$$

where $k = 1.0$. Then a sequence of transformations which could be used is

VI	OP	CI	CS	Interpretation
01	00	00	61	$x_1 \rightarrow$ CON(61), VAR(61)
01	02	61	62	$x_1^2 \rightarrow$ CON(62), VAR(62)
61	02	62	63	$x_1^3 \rightarrow$ CON(63), VAR(63)
01	00	00	14	$x_1 \rightarrow$ CON(14)
02	00	00	02	$x_2 \rightarrow$ CON(02)
02	02	61	12	$x_1x_2 \rightarrow$ CON(12)
03	00	00	11	$y_1 \rightarrow$ CON(11)
04	12	01	09	$\frac{k}{y_2} \rightarrow$ CON(09)

The TERMS information should then be

x_1	x_2	x_1x_2	x_1^3	y_1	$\frac{k}{y_2}$
14	02	12	63	11	09

The total process is illustrated by figure 4 and the type 4 input given in figure 5.

Type 5. - In type 5 input, one card with one one-column field and one three-column field specifies

(1) Use of t-statistics to reject insignificant terms (T or F)

(2) Probability level that t-statistic must meet to be considered significant (This is written without a decimal point; e.g., 95-percent significance level is supplied as 950, 99.9 percent as 999, etc. See line 23 of fig. 1.)

Type 6. - In type 6 input, one card with one one-column field specifies that the data contain replicated points (T or F). If there are replicated data points, as many cards as are needed are read, containing 20 four-column fields specifying

(1) The number of replicate sets

(2) The number of replicates in each replicate set

Note that it is not safe for the program to assume that all data points with the same levels of the independent variables are true replicates. For this reason, the user must arrange replicate sets. RAPIER does check that all independent terms are the same within a replicate set. If not, the program stops. A nonreplicated data point is considered to be a group of size 1. Note that the data in table I are grouped to clearly indicate the repli-

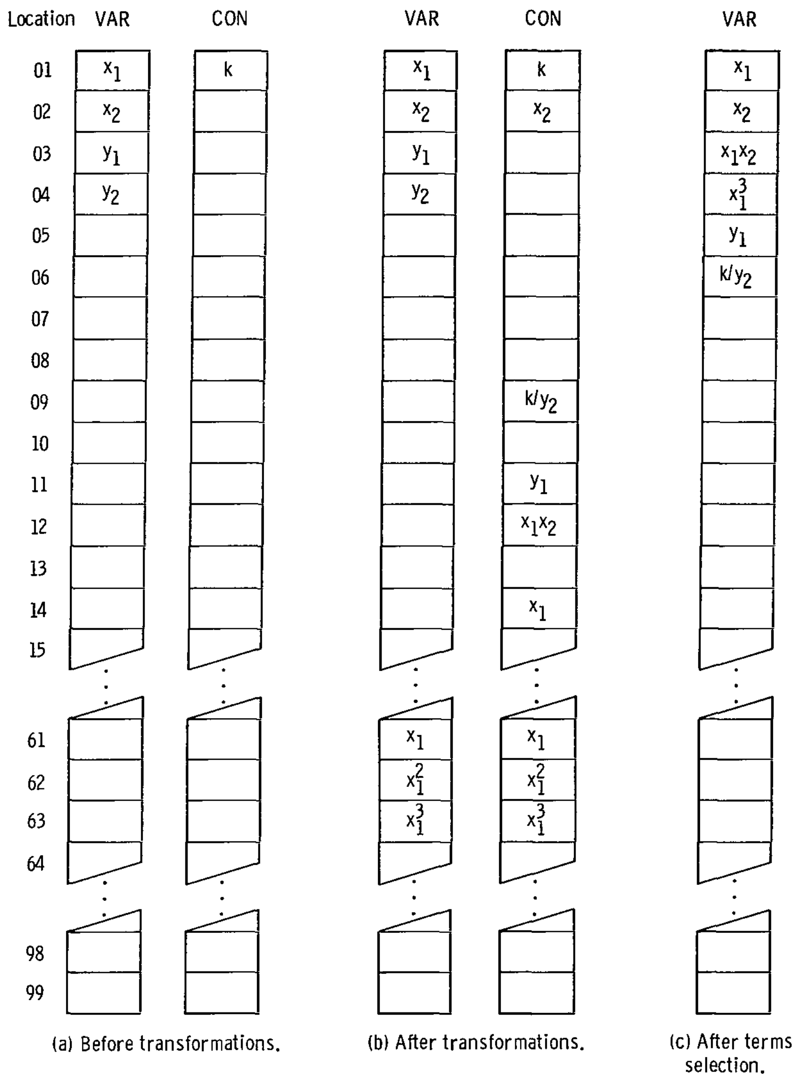


Figure 4. - Arrays VAR and CON before and after transformations and terms selection, for the second example.

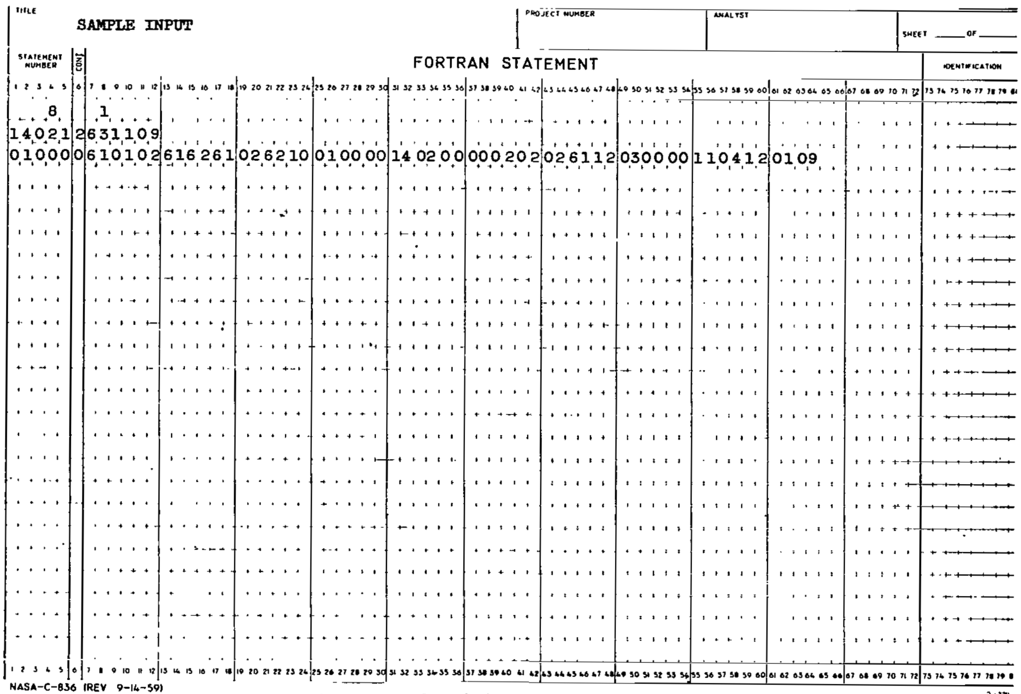


Figure 5. - Sample input form for type 4 input.

cated data points. There are 14 such groups. Thus, the first field of the second type 6 card contains a 14, and the remaining fields contain the count of the replicates in each group. (See lines 24 and 25 of fig. 1.)

Type 7. - In type 7 input, one card with one two-column field specifies the input unit number the data will be on. The remainder of the card is used to supply the format in which the data will be supplied. Note that if a weighting factor other than 1.0 is to be used, it will be read with each data point, and the format must allow for this. The current example uses a weighting factor of 1.0.

The format is (5F6.0) since there are three independent variables (x_1, x_2, x_3) and two dependent variables (y_1, y_2). If a weighting factor other than 1.0 is used, it must appear with every data point, and the format could, for example, be (5F6.0, F10.3). (See line 26 of fig. 1.)

Type 8. - Type 8 input consists of the input variables. Each observation consisting of the given x's and y's is read by execution of one READ statement. Thus, there will be at least one card for each observation. If the transformation option is not used, the program expects the first variables read to be the independent variables and the last ones to be the dependent variables; that is, the data must be arranged as

$$x_1, x_2, \ldots, x_J, \; y_1, y_2, \ldots, y_{NODEP}$$

Otherwise, if transformations are used, appropriate use of the terms card information allows for more flexibility of input. (See lines 27 to 50 of fig. 1.)

Type 9. - In type 9 input, one card with one column is used to indicate if predictions are desired (T or F). (See line 51 of fig. 1.) If this is false, a new case is started. If it is true, the following cards are read: One card with one four-column field specifies the number of predictions desired. This is followed by cards with the values of the independent variables at which predictions are desired. Only the final regression model is used, but the number of independent and dependent variables originally supplied on the type 2 data cards are read. All transformations in type 4 input are performed. Then the proper terms are chosen by the program to correspond to the final model. The dependent variables are not needed and, hence, may be left off the data card unless one of the transformations of the dependent variables might lead to an impossible operation (e.g., $\log(y_2)$). (See lines 52 and 53 of fig. 1.)

SAMPLE OUTPUT

```
 SAMPLE RAPIER PROBLEM
   MODEL EQUATION    I= 1,2

       Y(I) =  B0  +      B1*X1       +B2*X2       +B3*X3
                      + B12*X1*X2   +B13*X1*X3   +B23*X2*X3
                      + B11*X1**2   +B22*X2**2   +B33*X3**2       ERR

       X1 = TEMP             **
       X2 = TIME             ** DATA CODED TO STANDARIZED UNIT
       X3 = PRESS            **

       Y1 = POUNDS OUTPUT
       Y2 = COST OF OPERATION

       THE DATA IS FROM AN INCOMPLETE FACTORIAL DESIGN WITH  ONE
       REPLICATION (FICTITIOUS DATA)
   3   2   9   25
TTTTFFFFF2
THERE IS A B0 TO ESTIMATE
NTERM(K)=
  11  12  13  14  15  16  17  18  19  20  21
THE TRANSFORMATIONS ARE
   1   0   0  11        1   0   0  61        2   0   0  12        2   0   0  62        3   0   0  13
   3   0   0  63       61   2  61  17       62   2  62  18       63   2  63  19       61   2  62  14
  61   2  63  15       62   2  63  16        4   0   0  20        5   0   0  21




 ( 5F6.0)
 SAMPLE RAPIER PROBLEM

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,      1
 -1.000000     -1.000000     -1.000000      9.170000      38.50000
TERMS OF THE EQUATION, OBSERVATION =      1
 -1.000000     -1.000000     -1.000000      1.000000      1.000000      1.000000      1.000000      1.000000      1.000000
  9.170000      38.50000

** REPLICATE SET      1   ******************************************************************************************
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,      2
  1.000000     -1.000000     -1.000000      12.76000      43.10000
TERMS OF THE EQUATION, OBSERVATION =      2
  1.000000     -1.000000     -1.000000     -1.000000     -1.000000      1.000000      1.000000      1.000000      1.000000
  12.76000      43.10000

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,      3
  1.000000     -1.000000     -1.000000      12.97000      44.00000
TERMS OF THE EQUATION, OBSERVATION =      3
  1.000000     -1.000000     -1.000000     -1.000000     -1.000000      1.000000      1.000000      1.000000      1.000000
  12.97000      44.00000

** REPLICATE SET      2   ******************************************************************************************
  DEP. VAR.          1     SSQ= 0.2204895E-01     SUM= 25.730000        MEAN= 12.865000
  DEP. VAR.          2     SSQ= 0.4050598         SUM= 87.099999        MEAN= 43.550000
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,      4
 -1.000000      1.000000     -1.000000      9.110000      58.30000
TERMS OF THE EQUATION, OBSERVATION =      4
 -1.000000      1.000000     -1.000000     -1.000000      1.000000     -1.000000      1.000000      1.000000      1.000000
  9.110000      58.30000

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,      5
 -1.000000      1.000000     -1.000000      8.960000      58.70000
TERMS OF THE EQUATION, OBSERVATION =      5
 -1.000000      1.000000     -1.000000     -1.000000      1.000000     -1.000000      1.000000      1.000000      1.000000
  8.960000      58.70000

** REPLICATE SET      3   ******************************************************************************************
  DEP. VAR.          1     SSQ= 0.1125145E-01     SUM= 18.070000        MEAN= 9.0350000
  DEP. VAR.          2     SSQ= 0.7995605E-01     SUM= 117.00000        MEAN= 58.500000
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,      6
  1.000000      1.000000     -1.000000      17.03000      63.20000
TERMS OF THE EQUATION, OBSERVATION =      6
  1.000000      1.000000     -1.000000      1.000000     -1.000000     -1.000000      1.000000      1.000000      1.000000
  17.03000      63.20000
```

```
** REPLICATE SET     4   ************************************************************************************************
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,     7
 -1.000000     -1.000000      1.000000       9.050000        38.70000
TERMS OF THE EQUATION, OBSERVATION =      7
 -1.000000     -1.000000      1.000000       1.000000       -1.000000      -1.000000       1.000000       1.000000       1.000000
  9.050000      38.70000

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,     8
 -1.000000     -1.000000      1.000000       8.860000        39.60000
TERMS OF THE EQUATION, OBSERVATION =      8
 -1.000000     -1.000000      1.000000       1.000000       -1.000000      -1.000000       1.000000       1.000000       1.000000
  8.860000      39.60000

** REPLICATE SET     5   ************************************************************************************************
  DEP. VAR.        1    SSQ= 0.1805115E-01      SUM= 17.910000        MEAN= 8.9549999
  DEP. VAR.        2    SSQ= 0.4050293          SUM= 78.299999        MEAN= 39.150000
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,     9
  1.000000     -1.000000      1.000000      12.60000         44.10000
TERMS OF THE EQUATION, OBSERVATION =      9
  1.000000     -1.000000      1.000000      -1.000000        1.000000      -1.000000       1.000000       1.000000       1.000000
 12.60000       44.10000

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    10
  1.000000     -1.000000      1.000000      13.21000         43.80000
TERMS OF THE EQUATION, OBSERVATION =     10
  1.000000     -1.000000      1.000000      -1.000000        1.000000      -1.000000       1.000000       1.000000       1.000000
 13.21000       43.80000

** REPLICATE SET     6   ************************************************************************************************
  DEP. VAR.        1    SSQ= 0.1860542          SUM= 25.810000        MEAN= 12.905000
  DEP. VAR.        2    SSQ= 0.4501343E-01      SUM= 87.900000        MEAN= 43.950000
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    11
  1.000000      1.000000      1.000000      17.20000         62.10000
TERMS OF THE EQUATION, OBSERVATION =     11
  1.000000      1.000000      1.000000       1.000000        1.000000       1.000000       1.000000       1.000000       1.000000
 17.20000       62.10000

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    12
  1.000000      1.000000      1.000000      17.04000         62.80000
TERMS OF THE EQUATION, OBSERVATION =     12
  1.000000      1.000000      1.000000       1.000000        1.000000       1.000000       1.000000       1.000000       1.000000
 17.04000       62.80000

** REPLICATE SET     7   ************************************************************************************************
  DEP. VAR.        1    SSQ= 0.1280212E-01      SUM= 34.240000        MEAN= 17.120000
  DEP. VAR.        2    SSQ= 0.2449951          SUM= 124.90000        MEAN= 62.450000
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    13
  0             0             0              9.610000        49.10000
TERMS OF THE EQUATION, OBSERVATION =     13
  0             0             0              0               0              0               0              0              0
  9.610000      49.10000

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    14
  0             0             0             10.01000         49.30000
TERMS OF THE EQUATION, OBSERVATION =     14
  0             0             0              0               0              0               0              0              0
 10.01000       49.30000

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    15
  0             0             0             10.12000         50.10000
TERMS OF THE EQUATION, OBSERVATION =     15
  0             0             0              0               0              0               0              0              0
 10.12000       50.10000

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    16
  0             0             0              9.950000        51.80000
TERMS OF THE EQUATION, OBSERVATION =     16
  0             0             0              0               0              0               0              0              0
  9.950000      51.80000

** REPLICATE SET     8   ************************************************************************************************
  DEP. VAR.        1    SSQ= 0.1450768          SUM= 39.690000        MEAN= 9.9224999
  DEP. VAR.        2    SSQ= 4.5275879          SUM= 200.30000        MEAN= 50.074999
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    17
 -2.000000      0             0             11.78000         50.10000
TERMS OF THE EQUATION, OBSERVATION =     17
 -2.000000      0             0             -0              -0              0               4.000000       0              0
 11.78000       50.10000

** REPLICATE SET     9   ************************************************************************************************
*************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =   1.000000           OBSERVATION = ,    18
  2.000000      0             0             23.83000         57.60000
TERMS OF THE EQUATION, OBSERVATION =     18
  2.000000      0             0              0               0              0               4.000000       0              0
 23.83000       57.60000
```

```
OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,    19
  2.000000       0              0              22.90000       58.10000
TERMS OF THE EQUATION, OBSERVATION =     19
  2.000000       0              0              0              0              0              4.000000       0              0
  22.90000       58.10000

** REPLICATE SET     10  ***************************************************************************************************
  DEP. VAR.          1     SSQ= 0.4324341        SUM= 46.730000        MEAN= 23.365000
  DEP. VAR.          2     SSQ= 0.1250000        SUM= 115.70000        MEAN= 57.850000
************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,    20
  0             -2.000000       0              7.990000       28.60000
TERMS OF THE EQUATION, OBSERVATION =     20
  0             -2.000000       0             -0              0             -0              0              4.000000       0
  7.990000       28.60000

** REPLICATE SET     11  ***************************************************************************************************
************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,    21
  0              2.000000       0              12.11000       71.00000
TERMS OF THE EQUATION, OBSERVATION =     21
  0              2.000000       0              0              0              0              0              4.000000       0
  12.11000       71.00000

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,    22
  0              2.000000       0              11.70000       70.20000
TERMS OF THE EQUATION, OBSERVATION =     22
  0              2.000000       0              0              0              0              0              4.000000       0
  11.70000       70.20000

** REPLICATE SET     12  ***************************************************************************************************
  DEP. VAR.          1     SSQ= 0.8405304E-01    SUM= 23.810000        MEAN= 11.905000
  DEP. VAR.          2     SSQ= 0.3201904        SUM= 141.20000        MEAN= 70.599999
************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,    23
  0              0             -2.000000       10.11000       48.70000
TERMS OF THE EQUATION, OBSERVATION =     23
  0              0             -2.000000       0             -0             -0              0              0              4.000000
  10.11000       48.70000

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,    24
  0              0             -2.000000       10.01000       49.90000
TERMS OF THE EQUATION, OBSERVATION =     24
  0              0             -2.000000       0             -0             -0              0              0              4.000000
  10.01000       49.90000

** REPLICATE SET     13  ***************************************************************************************************
  DEP. VAR.          1     SSQ= 0.5002975E-02    SUM= 20.120000        MEAN= 10.060000
  DEP. VAR.          2     SSQ= 0.7200317        SUM= 98.599999        MEAN= 49.300000
************************************************************************************************************************

OBSERVED VARIABLES, WEIGHT =    1.000000          OBSERVATION = ,    25
  0              0              2.000000       10.02000       50.80000
TERMS OF THE EQUATION, OBSERVATION =     25
  0              0              2.000000       0              0              0              0              0              4.000000
  10.02000       50.80000

** REPLICATE SET     14  ***************************************************************************************************
************************************************************************************************************************

  SUMS OF INDEP AND DEP VARIABLES
   4.0000000      0              -2.0000000     0              2.0000000      -2.0000000     24.000000      24.000000
   24.000000      308.10000      1282.2000


X TRANSPOSE X MATRIX
ROW      1      24.00000
ROW      2      0              24.00000
ROW      3      2.000000      -2.000000       24.00000
ROW      4     -2.000000       2.000000       4.000000       12.00000
ROW      5      0              4.000000       2.000000      -2.000000       12.00000
ROW      6      4.000000       0             -2.000000       2.000000       0              12.00000
ROW      7      10.00000      -2.000000       0              0              2.000000      -2.000000       60.00000
ROW      8      2.000000       6.000000       0              0              2.000000      -2.000000       12.00000       60.00000
ROW      9      2.000000      -2.000000      -8.000000       0              2.000000      -2.000000       12.00000       12.00000


ROW      9      60.00000


X TRANSPOSE Y MATRIX
ROW      1      127.5600       260.5000
ROW      2      22.36000       238.5000
ROW      3     -12.24000      -110.3000
ROW      4      8.740000       12.90000
ROW      5      26.62000       139.7000
ROW      6     -9.680000      -95.90000
ROW      7      382.0000       1260.100
ROW      8      275.1600       1276.100
ROW      9      268.5200       1194.500
MEANS OF INDEP AND DEP VARIABLES
   0.1600000      0              -0.8000000E-01  0              0.8000000E-01 -0.8000000E-01  0.9600000      0.9600000
   0.9600000      12.324000      51.288000
```

```
X TRANSPOSE X MATRIX WHERE X IS DEVIATION FROM MEAN
ROW    1   23.36000
ROW    2  -0             24.00000
ROW    3   2.320000     -2.000000      23.84000
ROW    4  -2.000000      2.000000      4.000000      12.00000
ROW    5  -0.320000      4.000000      2.160000     -2.000000      11.84000
ROW    6   4.320000      0            -2.160000      2.000000      0.160000       11.84000
ROW    7   6.160000     -2.000000      1.920000     -0             0.800000E-01  -0.800000E-01   36.96000
ROW    8  -1.840000      6.000000      1.920000     -0             0.800000E-01  -0.800000E-01  -11.04000    36.96000
ROW    9  -1.840000     -2.000000     -6.080000     -0             0.800000E-01  -0.800000E-01  -11.04000   -11.04000


ROW    9   36.96000


X TRANSPOSE Y MATRIX WHERE X AND Y ARE DEVIATIONS FROM MEAN
ROW    1   78.26400      55.34800
ROW    2   22.36000      238.5000
ROW    3   12.40800     -7.723999
ROW    4   8.740000      12.90000
ROW    5   1.972000      37.12400
ROW    6   14.96800      6.676000
ROW    7   86.22400      29.18800
ROW    8  -20.61600      45.18800
ROW    9  -27.25600     -36.41200


CORRELATION COEFFICIENTS
ROW    1   1.000000
ROW    2  -0             1.000000
ROW    3   0.983102E-01 -0.836125E-01  1.000000
ROW    4  -0.119455      0.117851      0.236492      1.000000
ROW    5  -0.192414E-01  0.237289      0.128565     -0.167789      1.000000
ROW    6   0.259760      0            -0.128566      0.167789      0.135135E-01   1.000000
ROW    7   0.209642     -0.671519E-01  0.646818E-01 -0             0.382427E-02  -0.382427E-02   1.000000
ROW    8  -0.626203E-01  0.201456      0.646818E-01 -0             0.382427E-02  -0.382427E-02  -0.298701    1.000000
ROW    9  -0.626203E-01 -0.671519E-01 -0.204826     -0             0.382427E-02  -0.382427E-02  -0.298701   -0.298701


ROW    9   1.000000


 SAMPLE RAPIER PROBLEM
EACH COLUMN CONTAINS THE COEFFICIENTS FOR ONE DEPENDENT TERM
CONSTANT TERM (B0)
    9.973995       50.15123
REGRESSION COEFFICIENTS (B1,....,BK)
  1  2.940374      2.031365
  2  0.987377      10.16832
  3 -0.138332E-01  0.370808
  4  1.068721     -0.467807
  5  0.820396E-01 -0.397416
  6  0.203767E-01 -0.191481E-01
  7  1.909437      0.965708
  8  0.910342E-02 -0.571182E-01
  9  0.330273E-01 -0.596032E-03


 ANOVA OF REGRESSION ON DEPENDENT VARIABLE    1
*************************************************************************
        SOURCE          SUMS OF SQUARES    DEGREES OF FREEDOM  MEAN SQUARES
-------------------------------------------------------------------------
     REGRESSION         425.390369                 9          47.2655964
     RESIDUAL           1.08622742                15          0.72415160E-01
-------------------------------------------------------------------------
     TOTAL              426.476597                24
*************************************************************************
     R SQUARED = SSQ(REG) / SSQ(TOT) = 0.997453      R = .998726
     STANDARD ERROR OF ESTIMATE     0.269101
USING POOLING STRATEGY  2 THE ERROR MEAN SQUARE =  0.7241516E-01 WITH DEGREES OF FREEDOM =    15
    F=MS(REG)/MS(ERR)= 652.70     COMPARE TO F( 9, 15)


 ANOVA OF LACK OF FIT
*************************************************************************
        SOURCE          SUMS OF SQUARES    DEGREES OF FREEDOM  MEAN SQUARES
-------------------------------------------------------------------------
     LACK OF FIT        0.16945267                 5          0.33890533E-01
     REPLICATION        0.91677475                10          0.91677475E-01
     RESIDUAL           1.08622742                15          0.72415160E-01
-------------------------------------------------------------------------
     F = MS(LOF)/MS(REPS) =       0.370
*************************************************************************
```

ANOVA OF REGRESSION ON DEPENDENT VARIABLE 2

SOURCE	SUMS OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARES
REGRESSION	2539.42480	9	282.158310
RESIDUAL	10.4015808	15	0.69343872
TOTAL	2549.82639	24	

R SQUARED = SSQ(REG) / SSQ(TOT) = 0.995921 R = .997958
STANDARD ERROR OF ESTIMATE 0.832730
USING POOLING STRATEGY 2 THE ERROR MEAN SQUARE = 0.6934387 WITH DEGREES OF FREEDOM = 15
F=MS(REG)/MS(ERR)= 406.90 COMPARE TO F(9, 15)

ANOVA OF LACK OF FIT

SOURCE	SUMS OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARES
LACK OF FIT	3.52871704	5	0.70574340
REPLICATION	6.87286377	10	0.68728638
RESIDUAL	10.4015808	15	0.69343872

F = MS(LOF)/MS(REPS) = 1.027

SUMS OF SQUARES DUE TO EACH VARIABLE IF IT WERE LAST TO ENTER REGRESSION

1	164.5587	78.54016
2	19.45477	2063.286
3	0.349972E-02	2.514697
4	10.44572	2.001442
5	0.665367E-01	1.561362
6	0.405068E-02	0.357697E-02
7	96.39906	24.65779
8	0.221086E-02	0.870364E-01
9	0.288299E-01	0.938933E-05

STANDARD DEVIATION OF REGRESSION COEFFICIENTS (DERIVED FROM DIAGONAL ELEMENTS OF (X TRANSPOSE X)INVERSE MATRIX)

0	0.128439	0.397453
1	0.616818E-01	0.190874
2	0.602399E-01	0.186412
3	0.629246E-01	0.194720
4	0.889836E-01	0.275359
5	0.855870E-01	0.264848
6	0.861556E-01	0.266608
7	0.523340E-01	0.161947
8	0.521002E-01	0.161224
9	0.523440E-01	0.161978

(X TRANSPOSE X) INVERSE MATRIX

ROW	1	0.525393E-01							
ROW	2	-0.514411E-02	0.501117E-01						
ROW	3	-0.116754E-01	0.115598E-01	0.546779E-01					
ROW	4	0.191122E-01	-0.179948E-01	-0.285527E-01	0.109343				
ROW	5	0.892031E-02	-0.222912E-01	-0.193434E-01	0.307362E-01	0.101155			
ROW	6	-0.247187E-01	0.730194E-02	0.193819E-01	-0.311087E-01	-0.133644E-01	0.102503		
ROW	7	-0.892995E-02	0.149314E-02	0.198443E-02	-0.355785E-02	-0.223811E-02	0.471704E-02	0.378214E-01	
ROW	8	0.683955E-03	-0.771617E-02	-0.188310E-02	0.262664E-02	0.296380E-02	-0.619734E-03	0.152509E-01	0.374843E-01
ROW	9	-0.211930E-02	0.256242E-02	0.915151E-02	-0.513127E-02	-0.397529E-02	0.382758E-02	0.158305E-01	0.150502E-01

ROW 9 0.378359E-01
SAMPLE RAPIER PROBLEM

CALCULATED T STATISTICS
THE T STATISTICS CAN BE USED TO TEST THE NET REGRESSION COEFFICIENTS B(I).

47.67006	10.64246
16.39073	54.54760
0.219838	1.904315
12.01031	1.698899
0.958553	1.500541
0.236510	0.718214E-01
36.48561	5.963113
0.174729	0.354280
0.630967	0.367971E-02

UNDER NULL HYPOTHESIS THE INTERVAL (-T,T) WHERE T IS GIVEN ABOVE, HAS APPROX PROBABILITY LISTED BELOW.
MINUS SIGN INDICATES PROB EXCEEDS .999.

1	-0.999	-0.999
2	-0.999	-0.999
3	0.171	0.924
4	-0.999	0.890
5	0.647	0.845
6	0.184	0.056
7	-0.999	-0.999
8	0.135	0.271
9	0.463	0.002

THE DESIRED VALUE OF PROBABILITY IS 95.0 PERCENT
THE TERM X(8) IS BEING DELETED
THE NUMBER OF DEPENDENT VARIABLES WAS 2 IT IS BEING SET TO ONE AND THE REJECTION OPTION EXERCISED ON DEPENDENT VARIABLE 1

```
CORRELATION COEFFICIENTS
ROW    1    1.000000
ROW    2   -0              1.000000
ROW    3    0.983102E-01  -0.836125E-01   1.000000
ROW    4   -0.119455       0.117851       0.236492       1.000000
ROW    5   -0.192414E-01   0.237289       0.128566      -0.167789       1.000000
ROW    6    0.259760       0              -0.128565       0.167789       0.135135E-01   1.000000
ROW    7    0.209642      -0.671519E-01   0.546818E-01  -0              0.382427E-02  -0.382427E-02   1.000000
ROW    8   -0.626203E-01  -0.671519E-01  -0.234825      -0              0.382427E-02  -0.382427E-02  -0.298701      1.000000


 SAMPLE RAPIER PROBLEM
EACH COLUMN CONTAINS THE COEFFICIENTS FOR ONE DEPENDENT TERM
CONSTANT TERM (B0)
     9.989932
REGRESSION COEFFICIENTS (B1,....BK)
  1  2.940208
  2  0.989251
  3 -0.133747E-01
  4  1.068083
  5  0.813199E-01
  6  0.205272E-01
  7  1.905733
  9  0.293722E-01


 ANOVA OF REGRESSION ON DEPENDENT VARIABLE     1
****************************************************************************
        SOURCE           SUMS OF SQUARES     DEGREES OF FREEDOM  MEAN SQUARES
----------------------------------------------------------------------------
      REGRESSION         425.388165                  8            53.1735206
      RESIDUAL           1.08843231                 16            0.68027020E-01
----------------------------------------------------------------------------
      TOTAL              426.476597                 24
****************************************************************************
      R SQUARED = SSQ(REG) / SSQ(TOT) = 0.997448      R = .998723
      STANDARD ERROR OF ESTIMATE     0.260820
USING POOLING STRATEGY  2 THE ERROR MEAN SQUARE =  0.7241516E-01 WITH DEGREES OF FREEDOM =     15
    F=MS(REG)/MS(ERR)= 734.29      COMPARE TO F( 8, 15)

 ANOVA OF LACK OF FIT
****************************************************************************
        SOURCE           SUMS OF SQUARES     DEGREES OF FREEDOM  MEAN SQUARES
----------------------------------------------------------------------------
      LACK OF FIT        0.17165756                  6            0.28609593E-01
      REPLICATION        0.91677475                 10            0.91677475E-01
      RESIDUAL           1.08843231                 16            0.68027020E-01
----------------------------------------------------------------------------
      F = MS(LOF)/MS(REPS) =        0.312
****************************************************************************


 SUMS OF SQUARES DUE TO EACH VARIABLE IF IT WERE LAST TO ENTER REGRESSION
  1  164.5792
  2  20.16795
  3  0.327725E-02
  4  10.45084
  5  0.655261E-01
  6  0.411116E-02
  7  114.8713
  9  0.271357E-01


 STANDARD DEVIATION OF REGRESSION COEFFICIENTS (DERIVED FROM DIAGONAL ELEMENTS OF  (X TRANSPOSE X)INVERSE MATRIX )
  0  0.904301E-01
  1  0.616745E-01
  2  0.592775E-01
  3  0.628699E-01
  4  0.889087E-01
  5  0.854878E-01
  6  0.861513E-01
  7  0.478488E-01
  9  0.479823E-01


(X TRANSPOSE X) INVERSE MATRIX
ROW    1    0.525268E-01
ROW    2   -0.500332E-02   0.485234E-01
ROW    3   -0.116409E-01   0.111711E-01   0.545828E-01
ROW    4    0.190643E-01  -0.174541E-01  -0.284204E-01   0.109159
ROW    5    0.886623E-02  -0.216811E-01  -0.191941E-01   0.305285E-01   0.100920
ROW    6   -0.247074E-01   0.717437E-02   0.193507E-01  -0.310653E-01  -0.133154E-01   0.102493
ROW    7   -0.920822E-02   0.463255E-02   0.275263E-02  -0.462653E-02  -0.344396E-02   0.496919E-02   0.316154E-01
ROW    8   -0.239391E-02   0.566052E-02   0.990350E-02  -0.618589E-02  -0.516528E-02   0.407641E-02   0.970715E-02   0.317931E-01
```

```
 SAMPLE RAPIER PROBLEM

CALCULATED T STATISTICS
THE T STATISTICS CAN BE USED TO TEST THE NET REGRESSION COEFFICIENTS B(I).
  47.67303
  16.68845
  0.212736
  12.01326
  0.951245
  0.238269
  39.82824
  0.612147
UNDER NULL HYPOTHESIS THE INTERVAL (-T,T) WHERE T IS GIVEN ABOVE, HAS APPROX PROBABILITY LISTED BELOW.
 MINUS SIGN INDICATES PROB EXCEEDS .999.
  1         -0.999
  2         -0.999
  3          0.165
  4         -0.999
  5          0.644
  6          0.185
  7         -0.999
  9          0.451

THE DESIRED VALUE OF PROBABILITY IS  95.0 PERCENT
          THE TERM X( 3) IS BEING DELETED




CORRELATION COEFFICIENTS
ROW     1      1.000000
ROW     2    -0               1.000000
ROW     3    -0.119455        0.117851        1.000000
ROW     4    -0.192414E-01    0.237289       -0.167739        1.000000
ROW     5     0.259760        0               0.167739        0.135135E-01    1.000000
ROW     6     0.209642       -0.671519E-01   -0              0.382427E-02   -0.382427E-02    1.000000
ROW     7    -0.626203E-01   -0.671519E-01   -0              0.382427E-02   -0.382427E-02   -0.298701        1.000000


 SAMPLE RAPIER PROBLEM
EACH COLUMN CONTAINS THE COEFFICIENTS FOR ONE DEPENDENT TERM
CONSTANT TERM (B0)
     9.989235
REGRESSION COEFFICIENTS (B1,...,BK)
  1  2.937356
  2  0.991988
  4  1.061119
  5  0.766166E-01
  6  0.252688E-01
  7  1.906408
  9  0.318004E-01



 ANOVA OF REGRESSION ON DEPENDENT VARIABLE     1
******************************************************************************
      SOURCE           SUMS OF SQUARES     DEGREES OF FREEDOM  MEAN SQUARES
------------------------------------------------------------------------------
    REGRESSION         425.384892                  7             60.7692695
    RESIDUAL           1.09170532                 17             0.64217960E-01
------------------------------------------------------------------------------
    TOTAL              426.476597                 24
******************************************************************************
       R SQUARED = SSQ(REG) / SSQ(TOT) = 0.997440       R = .998719
       STANDARD ERROR OF ESTIMATE      0.253413
USING POOLING STRATEGY  2 THE ERROR MEAN SQUARE =  0.7241516E-01 WITH DEGREES OF FREEDOM =     15
     F=MS(REG)/MS(ERR)= 839.18      COMPARE TO F( 7, 15)

 ANOVA OF LACK OF FIT
******************************************************************************
      SOURCE           SUMS OF SQUARES     DEGREES OF FREEDOM  MEAN SQUARES
------------------------------------------------------------------------------
    LACK OF FIT        0.17493057                  7             0.24990082E-01
    REPLICATION        0.91677475                 10             0.91677475E-01
    RESIDUAL           1.09170532                 17             0.64217960E-01
------------------------------------------------------------------------------
    F = MS(LOF)/MS(REPS) =        0.273
******************************************************************************


 SUMS OF SQUARES DUE TO EACH VARIABLE IF IT WERE LAST TO ENTER REGRESSION
  1  172.4090
  2  21.28251
  4  11.93265
  5  0.623348E-01
  6  0.667668E-02
  7  115.4596
  9  0.337156E-01
```

```
STANDARD DEVIATION OF REGRESSION COEFFICIENTS (DERIVED FROM DIAGONAL ELEMENTS OF  (X TRANSPOSE X)INVERSE MATRIX )
 0  0.903708E-01
 1  0.601993E-01
 2  0.578642E-01
 4  0.826629E-01
 5  0.825796E-01
 6  0.832182E-01
 7  0.477436E-01
 9  0.466049E-01


(X TRANSPOSE X) INVERSE MATRIX
ROW     1      0.500441E-01
ROW     2     -0.262083E-02   0.462370E-01
ROW     3      0.130030E-01  -0.116374E-01   0.943608E-01
ROW     4      0.477268E-02  -0.177527E-01   0.205345E-01   0.941707E-01
ROW     5     -0.205804E-01   0.321397E-02  -0.209897E-01  -0.651070E-02   0.956329E-01
ROW     6     -0.862117E-02   0.406919E-02  -0.319328E-02  -0.247600E-02   0.399332E-02   0.314776E-01
ROW     7     -0.280475E-03   0.363238E-02  -0.102611E-02  -0.168055E-02   0.563249E-03   0.920740E-02   0.299940E-01


 SAMPLE RAPIER PROBLEM

CALCULATED T STATISTICS
THE T STATISTICS CAN BE USED TO TEST THE NET REGRESSION COEFFICIENTS B(I).
  48.79386
  17.14339
  12.83671
  0.927792
  0.303645
  39.93009
  0.682340
UNDER NULL HYPOTHESIS THE INTERVAL (-T,T) WHERE T IS GIVEN ABOVE, HAS APPROX PROBABILITY LISTED BELOW.
 MINUS SIGN INDICATES PROB EXCEEDS .999.
  1         -0.999
  2         -0.999
  4         -0.999
  5          0.632
  6          0.233
  7         -0.999
  9          0.495

THE DESIRED VALUE OF PROBABILITY IS  95.0 PERCENT
          THE TERM X( 6) IS BEING DELETED




CORRELATION COEFFICIENTS
ROW     1      1.000000
ROW     2     -0              1.000000
ROW     3     -0.119455       0.117851        1.000000
ROW     4     -0.192414E-01   0.237289       -0.167789        1.000000
ROW     5      0.209642      -0.671519E-01   -0              0.382427E-02   1.000000
ROW     6     -0.626203E-01  -0.671519E-01   -0              0.382427E-02  -0.298701        1.000000


 SAMPLE RAPIER PROBLEM
EACH COLUMN CONTAINS THE COEFFICIENTS FOR ONE DEPENDENT TERM
CONSTANT TERM (B0)
    9.987362
REGRESSION COEFFICIENTS (B1,...,BK)
  1  2.942794
  2  0.991139
  4  1.066665
  5  0.783369E-01
  7  1.905353
  9  0.316516E-01


 ANOVA OF REGRESSION ON DEPENDENT VARIABLE     1
******************************************************************************
     SOURCE            SUMS OF SQUARES     DEGREES OF FREEDOM  MEAN SQUARES
------------------------------------------------------------------------------
    REGRESSION         425.378220                 5               70.8963690
    RESIDUAL           1.09837723                18               0.61020957E-01
------------------------------------------------------------------------------
    TOTAL              426.476597                24
******************************************************************************
      R SQUARED = SSQ(REG) / SSQ(TOT) = 0.997425       R = .998711
       STANDARD ERROR OF ESTIMATE     0.247024
USING POOLING STRATEGY  2 THE ERROR MEAN SQUARE =  0.7241516E-01 WITH DEGREES OF FREEDOM =      15
    F=MS(REG)/MS(ERR)= 979.03      COMPARE TO F( 6, 15)
```

```
 ANOVA OF LACK OF FIT
********************************************************************************
      SOURCE           SUMS OF SQUARES      DEGREES OF FREEDOM  MEAN SQUARES
--------------------------------------------------------------------------------
    LACK OF FIT        0.18160248                  8              0.22700310E-01
    REPLICATION        0.91677475                 10              0.91677475E-01
    RESIDUAL           1.09837723                 18              0.61020957E-01
--------------------------------------------------------------------------------
    F = MS(LOF)/MS(REPS) =        0.248
********************************************************************************


 SUMS OF SQUARES DUE TO EACH VARIABLE IF IT WERE LAST TO ENTER REGRESSION
   1  189.8499
   2  21.29583
   4  12.67660
   5  0.654736E-01
   7  115.9460
   9  0.334045E-01


 STANDARD DEVIATION OF REGRESSION COEFFICIENTS (DERIVED FROM DIAGONAL ELEMENTS OF  (X TRANSPOSE X)INVERSE MATRIX )
   0  0.901600E-01
   1  0.574737E-01
   2  0.577965E-01
   4  0.806197E-01
   5  0.823850E-01
   7  0.476170E-01
   9  0.466024E-01


(X TRANSPOSE X) INVERSE MATRIX
ROW      1       0.456152E-01
ROW      2      -0.192918E-02   0.461290E-01
ROW      3       0.848598E-02  -0.109320E-01   0.897539E-01
ROW      4       0.337156E-02  -0.175339E-01   0.191055E-01   0.937275E-01
ROW      5      -0.776179E-02   0.393498E-02  -0.231582E-02  -0.220413E-02   0.313109E-01
ROW      6      -0.159262E-03   0.361345E-02  -0.902437E-03  -0.164221E-02   0.918388E-02   0.299907E-01
 SAMPLE RAPIER PROBLEM

CALCULATED T STATISTICS
THE T STATISTICS CAN BE USED TO TEST THE NET REGRESSION COEFFICIENTS B(I).
  51.20241
  17.14875
  13.23082
  0.950864
  40.01411
  0.679185
UNDER NULL HYPOTHESIS THE INTERVAL (-T,T) WHERE T IS GIVEN ABOVE, HAS APPROX PROBABILITY LISTED BELOW.
 MINUS SIGN INDICATES PROB EXCEEDS .999.
  1          -0.999
  2          -0.999
  4          -0.999
  5           0.643
  7          -0.999
  9           0.493

THE DESIRED VALUE OF PROBABILITY IS  95.0 PERCENT
          THE TERM X( 9) IS BEING DELETED




CORRELATION COEFFICIENTS
ROW      1       1.000000
ROW      2      -0               1.000000
ROW      3      -0.119455        0.117851         1.000000
ROW      4      -0.192414E-01    0.237289        -0.167739         1.000000
ROW      5       0.209642       -0.671519E-01    -0               0.382427E-02   1.000000


 SAMPLE RAPIER PROBLEM
EACH COLUMN CONTAINS THE COEFFICIENTS FOR ONE DEPENDENT TERM
CONSTANT TERM (B0)
    10.02689
REGRESSION COEFFICIENTS (B1,....,BK)
  1  2.942962
  2  0.987325
  4  1.067618
  5  0.800701E-01
  7  1.895660
```

```
 ANOVA OF REGRESSION ON DEPENDENT VARIABLE     1
*******************************************************************************
      SOURCE          SUMS OF SQUARES     DEGREES OF FREEDOM   MEAN SQUARES
-------------------------------------------------------------------------------
    REGRESSION        425.344822                  5              85.0689640
    RESIDUAL          1.13177490                 19              0.59567100E-01
-------------------------------------------------------------------------------
    TOTAL             426.476597                 24
*******************************************************************************
    R SQUARED = SSQ(REG) / SSQ(TOT) = 0.997345       R = .998672
     STANDARD ERROR OF ESTIMATE     0.244054
USING POOLING STRATEGY  2 THE ERROR MEAN SQUARE =  0.7241516E-01 WITH DEGREES OF FREEDOM =     15
    F=MS(REG)/MS(ERR)= 174.74      COMPARE TO F( 5, 15)


 ANOVA OF LACK OF FIT
*******************************************************************************
      SOURCE          SUMS OF SQUARES     DEGREES OF FREEDOM   MEAN SQUARES
-------------------------------------------------------------------------------
    LACK OF FIT       0.21500015                  9              0.23888906E-01
    REPLICATION       0.91677475                 10              0.91677475E-01
    RESIDUAL          1.13177490                 19              0.59567100E-01
-------------------------------------------------------------------------------
    F = MS(LOF)/MS(REPS) =        0.261
*******************************************************************************


 SUMS OF SQUARES DUE TO EACH VARIABLE IF IT WERE LAST TO ENTER REGRESSION
  1  189.8751
  2  21.33362
  4  12.70310
  5  0.684685E-01
  7  126.0952


 STANDARD DEVIATION OF REGRESSION COEFFICIENTS (DERIVED FROM DIAGONAL ELEMENTS OF  (X TRANSPOSE X)INVERSE MATRIX )
  0  0.688638E-01
  1  0.574732E-01
  2  0.575232E-01
  4  0.806076E-01
  5  0.823455E-01
  7  0.454282E-01


(X TRANSPOSE X) INVERSE MATRIX
ROW      1       0.456143E-01
ROW      2      -0.190999E-02    0.456936E-01
ROW      3       0.848119E-02   -0.108233E-01    0.897258E-01
ROW      4       0.336284E-02   -0.173360E-01    0.190551E-01    0.935375E-01
ROW      5      -0.771302E-02    0.282846E-02   -0.204045E-02   -0.170125E-02    0.284985E-01
 SAMPLE RAPIER PROBLEM

CALCULATED T STATISTICS
THE T STATISTICS CAN BE USED TO TEST THE NET REGRESSION COEFFICIENTS B(I).
   51.20581
   17.16396
   13.24464
   0.972368
   41.72868
UNDER NULL HYPOTHESIS THE INTERVAL (-T,T) WHERE T IS GIVEN ABOVE, HAS APPROX PROBABILITY LISTED BELOW.
 MINUS SIGN INDICATES PROB EXCEEDS .999.
  1         -0.999
  2         -0.999
  4         -0.999
  5          0.654
  7         -0.999

THE DESIRED VALUE OF PROBABILITY IS  95.0 PERCENT
         THE TERM X( 5) IS BEING DELETED





CORRELATION COEFFICIENTS
ROW     1     1.000000
ROW     2    -0                 1.000000
ROW     3    -0.119455          0.117851        1.000000
ROW     4     0.209642         -0.671519E-01   -0                 1.000000


 SAMPLE RAPIER PROBLEM
EACH COLUMN CONTAINS THE COEFFICIENTS FOR ONE DEPENDENT TERM
CONSTANT TERM (B0)
     10.03236
REGRESSION COEFFICIENTS (B1,....,BK)
  1  2.940086
  2  1.002149
  4  1.051323
  7  1.897115
```

```
 ANOVA OF REGRESSION ON DEPENDENT VARIABLE     1
******************************************************************************
       SOURCE            SUMS OF SQUARES      DEGREES OF FREEDOM  MEAN SQUARES
------------------------------------------------------------------------------
     REGRESSION         425.276363                 4              106.319091
     RESIDUAL           1.20023346                 20             0.60011672E-01
------------------------------------------------------------------------------
     TOTAL              426.476597                 24
******************************************************************************
     R SQUARED = SSQ(REG) / SSQ(TOT) = 0.997185       R = .998592
     STANDARD ERROR OF ESTIMATE      0.244973
USING POOLING STRATEGY  2 THE ERROR MEAN SQUARE =  0.7241516E-01 WITH DEGREES OF FREEDOM =     15
     F=MS(REG)/MS(ERR)= 468.19      COMPARE TO F( 4, 15)

 ANOVA OF LACK OF FIT
******************************************************************************
       SOURCE            SUMS OF SQUARES      DEGREES OF FREEDOM  MEAN SQUARES
------------------------------------------------------------------------------
     LACK OF FIT        0.28345871                 10             0.28345871E-01
     REPLICATION        0.91677475                 10             0.91677475E-01
     RESIDUAL           1.20023346                 20             0.60011672E-01
------------------------------------------------------------------------------
     F = MS(LOF)/MS(REPS) =       0.309
******************************************************************************


 SUMS OF SQUARES DUE TO EACH VARIABLE IF IT WERE LAST TO ENTER REGRESSION
  1  190.0073
  2  23.63953
  4  12.87474
  7  126.4260


 STANDARD DEVIATION OF REGRESSION COEFFICIENTS (DERIVED FROM DIAGONAL ELEMENTS OF  (X TRANSPOSE X)INVERSE MATRIX )
  0  0.686337E-01
  1  0.573971E-01
  2  0.554661E-01
  4  0.788463E-01
  7  0.454036E-01


(X TRANSPOSE X) INVERSE MATRIX
ROW     1      0.454936E-01
ROW     2     -0.128740E-02   0.424841E-01
ROW     3      0.779683E-02  -0.729524E-02   0.858487E-01
ROW     4     -0.765192E-02   0.251349E-02  -0.169424E-02    0.284676E-01
 SAMPLE RAPIER PROBLEM

CALCULATED T STATISTICS
THE T STATISTICS CAN BE USED TO TEST THE NET REGRESSION COEFFICIENTS B(I).
   51.22363
   18.06778
   13.33382
   41.78336
UNDER NULL HYPOTHESIS THE INTERVAL (-T,T) WHERE T IS GIVEN ABOVE, HAS APPROX PROBABILITY LISTED BELOW.
 MINUS SIGN INDICATES PROB EXCEEDS .999.
   1          -0.999
   2          -0.999
   4          -0.999
   7          -0.999

THE DESIRED VALUE OF PROBABILITY IS   95.0 PERCENT
 SAMPLE RAPIER PROBLEM
FOR EACH DEPENDENT TERM AND OBSERVATION IS PRINTED
OBSERVED RESPONSE (Y OBSERVED)
CALCULATED RESPONSE (Y CALC)
RESIDUAL (YOBS- YCALC=YDIF)
STANDARDIZED RESIDUAL ( Z )

Y OBSERVED      9.1700
Y CALC          9.0386
Y DIF           0.1314
STUDENTIZED     0.4884

Y OBSERVED      12.760
Y CALC          12.816
Y DIF          -0.5608E-01
STUDENTIZED    -0.2084

Y OBSERVED      12.970
Y CALC          12.816
Y DIF           0.1539
STUDENTIZED     0.5720

Y OBSERVED      9.1100
Y CALC          8.9402
Y DIF           0.1698
STUDENTIZED     0.6310

Y OBSERVED      8.9600
Y CALC          8.9402
Y DIF           0.1979E-01
STUDENTIZED     0.7354E-01
```

Y OBSERVED 17.030
Y CALC 16.923
Y DIF 0.1070
STUDENTIZED 0.3975

Y OBSERVED 9.0500
Y CALC 9.0386
Y DIF 0.1144E-01
STUDENTIZED 0.4252E-01

Y OBSERVED 8.8600
Y CALC 9.0386
Y DIF -0.1786
STUDENTIZED -0.6635

Y OBSERVED 12.600
Y CALC 12.816
Y DIF -0.2161
STUDENTIZED -0.8030

Y OBSERVED 13.210
Y CALC 12.816
Y DIF 0.3939
STUDENTIZED 1.4638

Y OBSERVED 17.200
Y CALC 16.923
Y DIF 0.2770
STUDENTIZED 1.0292

Y OBSERVED 17.040
Y CALC 16.923
Y DIF 0.1170
STUDENTIZED 0.4347

Y OBSERVED 9.6100
Y CALC 10.032
Y DIF -0.4224
STUDENTIZED -1.5695

Y OBSERVED 10.010
Y CALC 10.032
Y DIF -0.2236E-01
STUDENTIZED -0.8308E-01

Y OBSERVED 10.120
Y CALC 10.032
Y DIF 0.8764E-01
STUDENTIZED 0.3257

Y OBSERVED 9.9500
Y CALC 10.032
Y DIF -0.8236E-01
STUDENTIZED -0.3060

Y OBSERVED 11.780
Y CALC 11.741
Y DIF 0.3936E-01
STUDENTIZED 0.1463

Y OBSERVED 23.830
Y CALC 23.501
Y DIF 0.3290
STUDENTIZED 1.2226

Y OBSERVED 22.900
Y CALC 23.501
Y DIF -0.6010
STUDENTIZED -2.2333

Y OBSERVED 7.9900
Y CALC 8.0281
Y DIF -0.3806E-01
STUDENTIZED -0.1414

Y OBSERVED 12.110
Y CALC 12.037
Y DIF 0.7335E-01
STUDENTIZED 0.2726

Y OBSERVED 11.700
Y CALC 12.037
Y DIF -0.3367
STUDENTIZED -1.2510

Y OBSERVED 10.110
Y CALC 10.032
Y DIF 0.7764E-01
STUDENTIZED 0.2885

Y OBSERVED 10.010
Y CALC 10.032
Y DIF -0.2236E-01
STUDENTIZED -0.8308E-01

Y OBSERVED 10.020
Y CALC 10.032
Y DIF -0.1236E-01
STUDENTIZED -0.4592E-01

```
SAMPLE RAPIER PROBLEM

         SKEWNESS (SHOULD BE NEAR ZERO)

                 0.1928
         KURTOSIS (SHOULD BE NEAR THREE)

                 1.6949

CHI-SQUARE IS NOT COMPUTED FOR LESS THAN 30 DEGREES OF FREEDOM FOR ERROR.


FOR EACH SET OF INDEP VARIABLES THERE IS COMPUTED...
PREDICTED RESPONSE
VARIANCE OF REGRESSION LINE
STANDARD DEVIATION OF REGRESSION
VARIANCE OF PREDICTED VALUE
STANDARD DEVIATION OF PREDICTED VALUE

INPUT DATA FOR THIS PREDICTED RESPONSE
 -1.000000      1.000000      1.000000      -0            -0

 INDEPENDENT TERMS ACCORDING TO FINAL REGRESSION MODEL
 -1.000000      1.000000     -1.000000      1.000000

PREDICTED RESPONSE FOR ABOVE INDEP VARIABLES
  8.940211
  0.192847E-01
  0.138869
  0.916998E-01
  0.302820
```

Lewis Research Center,

National Aeronautics and Space Administration,

Cleveland, Ohio, October 16, 1969,

129-04.

APPENDIX A

PROGRAM DOCUMENTATION AND LISTINGS

The contents of this appendix include a flow chart of the program, a listing of the routines used in RAPIER and their major functions, the call structure of the program, a dictionary of the program, and the listing.

General Mathematical and Logical Flow of Program

The flow of operation in RAPIER is illustrated in figure 6.

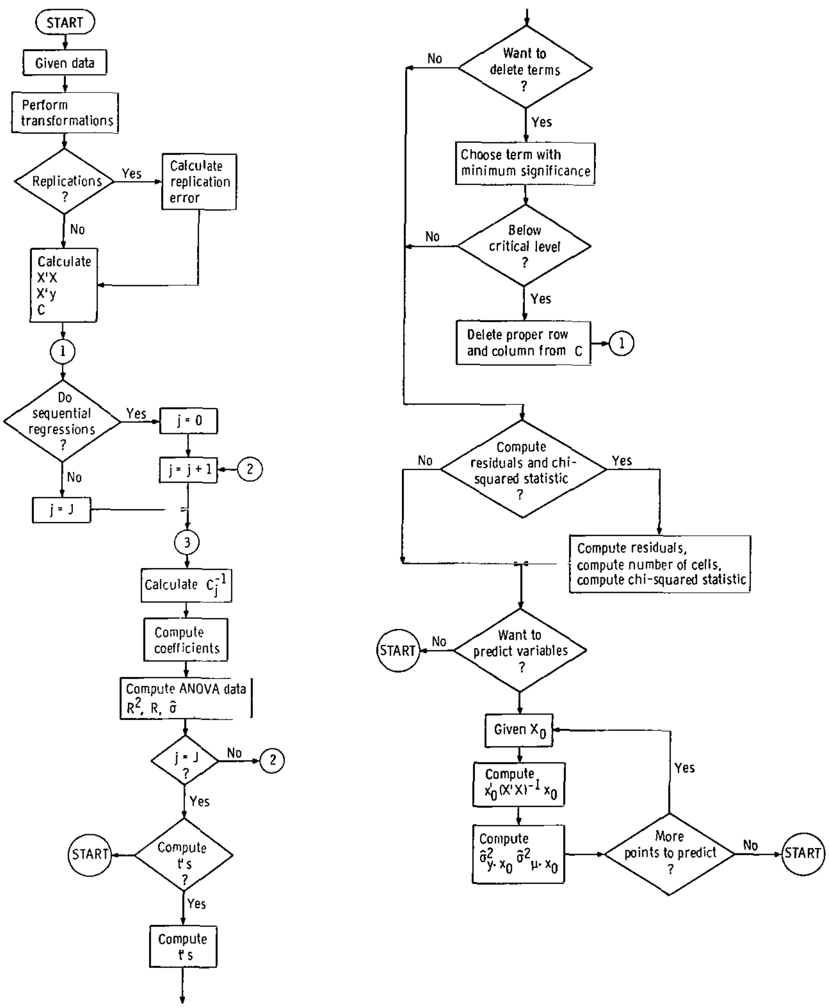


Figure 6. - Flow chart for RAPIER.

Routines and Their Major Functions

FORTRAN name	Function of routine
BORD	Inverts symmetric matrix A of order n by adding bordering column to already inverted matrix of order n - 1
CHISQ	Computes residuals at observed points and chi-square statistic to test goodness of fit
HIST	Prints histogram of residuals
INVXTX	Inverts symmetric matrix by Gauss elimination
LOC	When given row and column indices of symmetric matrix element, it computes location this element would have if only upper triangular part were stored as vector.
MATINV	Controls inversion process; computes regression coefficients; computes eigenvalues and eigenvectors of X'X if requested
MFIX	Prints and truncates X'X and computes C
PREDCT	Computes predicted values, variances, and standard deviations of regression line and further observations at specified points
RAPIER	Executes overall problem control; computes replication error; controls deletion of variables when given results of t-tests; controls most input and output
RECT	Writes rectangular matrix
RSTATS	Computes regression statistics and writes regression and lack-of-fit analysis of variance tables
SUMUPS	Constructs X'X and X'y matrices one observation at a time, in double precision
TRAN	Performs transformations
TRIANG	Writes lower triangular part of symmetric matrix
TTEST	Computes t-statistics and their significance levels; determines which variable should be deleted
EIGEN	Computes eigenvalues and eigenvectors of input symmetric matrix

Call Structure of Program

The call structure of the program is illustrated in figure 7.

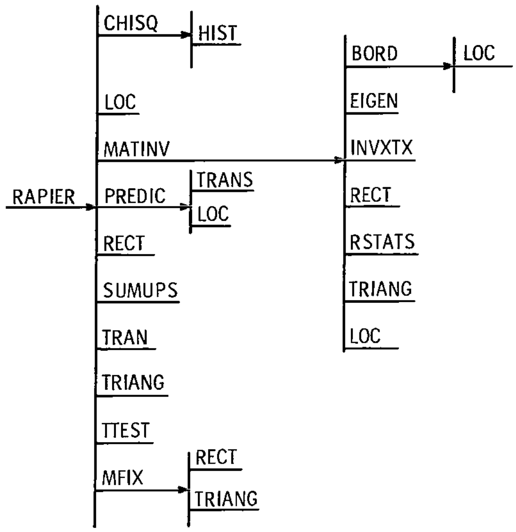


Figure 7. - Call structure of RAPIER.

Dictionary of Program

FORTRAN name	Mathematical symbol	Description
ACTDEV	e_i	Error in observation i; difference between observed and predicted response
ZEAN	$E(X)$, μ	Expected or average value (of X)
B		Regression coefficients other than the constant
BO		Constant regression coefficient
CHI	χ^2	Chi-squared statistic
CON		Constants used in transformations, and results of transformations
CORR	C	Correlation matrix

FORTRAN name	Mathematical symbol	Description
DELETE		Logical variable set to TRUE when deletion of terms is desired
ERRMS	$\hat{\sigma}^2$	Estimate of σ^2 used in hypothesis tests
EIG		Overflow area for storing part of modal matrix
FMT		Variable input format
FMTTRI		Format for printing matrix
IDENT		First identification printed at top of each page
IDOUT		Original sequence number of each term relating reduced models to original model
IFCHI		Logical variable set to TRUE if chi-square option is desired
IFSSR		Logical variable set to TRUE if sequential regressions are desired
IFTT		Logical variable set to TRUE if t-statistics are desired
IFWT		Logical variable set to TRUE if all weights of observations are 1.0
INPUT		Input logical tape unit number for data
INPUT5		Set equal to 5 to denote input device is card reader
INTER		Tape unit where input data is stored for chi-square or prediction routines
IOUT		Sequence number of term among those remaining which is to be deleted
JCOL		Total number of independent and dependent terms in regression model
KONNO		Number of constants originally supplied for transformations
LENGTH		Number of locations in correlation matrix storage area currently needed
LIST		Set equal to 6 to denote output device is printer

FORTRAN name	Mathematical symbol	Description
NARAY		Number of replications per replicate set
NCON		Array containing addresses in CON array for use in transformations
NERROR		Degrees of freedom for error mean square estimate
NODEP		Number of dependent variables
NOOB	N	Number of observations
BZERO		Logical variable set to TRUE if constant b_0 coefficient should be in regression model
NOTERM	J	Number of terms in current regression model
NOVAR	K	Number of independent variables to be read
NPDEG	NPDEG	Pooled degrees of freedom for replication error
NRES	N - J - D	Degrees of freedom for estimation of residual variance
NTERM		Array containing locations of terms in CON array that should be in regression model
NTRAN		Array containing transformation codes for use in performing transformations
NTRANS		Number of transformations to perform
NWHERE		Location in X array of first dependent variable; used in prediction routine to adjust for deleted terms
NXCOD		Array containing addresses of variables (or terms with address > 60) for use in transformations
P		Probability that the interval (-t, t) must have before a term is considered to be significant
POOLED	SSQ(REP)	Array containing pooled sums of squares from replications for each dependent term
PREDCT		Logical variable set to TRUE if prediction option is desired
RELSKW	RELSKW	Skewness of distribution of residuals

FORTRAN name	Mathematical symbol	Description
RELKUR	RELKUR	Kurtosis of distribution of residuals
REPS		Logical variable set to TRUE if there are replicate sets in the data
REPVAR		Array containing replication variance of each dependent term
RESMS		Array containing residual mean square or variance of each dependent term
RWT		Reciprocal of total weight
SUMX	$\sum x, \sum y$	Array containing sums of independent and dependent terms
SUMX2	$\sum x^2, \sum y^2$	Array containing sum of squared independent and dependent terms
SUMXX	$X'X$	Sums of squares and crossproducts matrix, and variance-covariance matrix of independent terms
SUMXXI	$(X'X)^{-1}$	Variance-covariance matrix of estimated regression coefficients
SUMXY	$X'y$	Array containing sums of crossproducts of independent terms with dependent terms
TOTWT	$\sum w_i$	Sum of weight of observations
X		Before transformations are performed, this contains the variables as read in. After the transformations are performed, appropriate data from the CON array are placed here according to information on the terms cards.
NLOF	N - J - NPDEG - D	Degrees of freedom for estimating variance due to lack of fit
NREG	J	Degrees of freedom for determining variance due to regression
NTOT	N-D	Total degrees of freedom
RNLOF		Reciprocal of degrees of freedom for lack of fit
RNREG		Reciprocal of degrees of freedom for regression

FORTRAN name	Mathematical symbol	Description
RNRES		Reciprocal of degrees of freedom for residual
STORYI		Logical variable set to TRUE if product $C \cdot C^{-1}$ is to be computed and printed
STORYC		Logical variable set to TRUE if eigenvectors and eigenvalues of C are to be computed and printed
STORYX		Logical variable set TRUE if eigenvectors and eigenvalues of $X'X$ are to be computed and printed
SATRTD		Logical variable indicating that there are no degrees of freedom for residual if TRUE
ECONMY		Logical variable indicating suppress printout of $x'x$, $x'x$ deviations, and C if TRUE
XCHK		Array used in checking if all values of independent terms are the same within a replicate set

Program Listing

```
$IBFTC BLCV

      BLCCK DATA
      COMMCN/SMALL/     BYPASS,BZERO,DELETE, FIRST, IFCHI,  IFSSR,
     X      IFTT,       IFWT,        INPUT,       INPUT5,      INTER,
     X      ISTRAT,     JCOL,        KONNO,       LENGTH,      LIST,
     X      NERRCR,     NODEP,       NOOB,                     NOTERM,
     X      NOVAR,      NPBEG,       NRES,        NTRANS,      NWHERE,
     X      P,          PREDCT,      REPS,        RWT,
     X      STORYI,      STORYC,      STORYX,      TOTWT,       WEIGHT,
     X      ERRFXD, ECONMY
      LOGICAL ECCNMY
      DCUBLE PRECISION RWT,TOTWT,WEIGHT
      COMMCN /FRMTS/ FMT(13),FMTTRI(14)
      DATA  INTER/3/,INPUT5/5/, LIST/6/
      DATA (FMTTRI(I),I=1,4)/6H(5H RO, 6HW I5,2, 6HX,(8G1, 6H5.6)) /
      ENC
```

```
$IBFTC RAPIER

C
C     THIS IS RAPIER,MAIN PROGRAM FOR REGRESSION ANALYSIS PROVIDING
C     INTERNAL EVALUATION OF RESULTS.
C***********************************************************************
C
      COMMON/BIG/SUMXX(1830),SUMXXI(1830),EIG(1830),SUMXY(60,9),
     X  B(60,9),CORR(1830)
      COMMON/MED/         BO(9),            CON(99),           ERRMS(9),
     X   IDENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                            RESMS(9),          SUMX(138),
     XSUMX2(69),          X(99),            ZEAN(69),          SUMY2(18)
        COMMON /FRMTS/ FMT(13),FMTTRI(14)
      COMMON/SMALL/       BYPASS,BZERO,DELETE, FIRST, IFCHI,   IFSSR,
     X       IFTT,        IFWT,       INPUT,       INPUT5,     INTER,
     X       ISTRAT,      JCOL,       KONNO,       LENGTH,     LIST,
     X       NERROR,      NODEP,      NOOB,                    NOTERM,
     X       NOVAR,       NPDEG,      NRES,        NTRANS,     NWHERE,
     X       P,           PREDCT,     REPS,        RWT,
     X       STORYI,       STORYC,      STORYX,      TOTWT,      WEIGHT,
     X       ERRFXD, ECONMY
      LOGICAL ECONMY
      DOUBLE PRECISION RWT,TOTWT,WEIGHT
      LOGICAL      BYPASS,     BZERO,      DELETE,     IFCHI,
     XIFSSR,      IFTT,       IFWT,       REPS,       PREDCT,
     XSTORYC,     STORYX,     STORYI,     FIRST ,ERRFXD
      LOGICAL XSAVE
      DIMENSION XCHK(60)
      COMMON/CNTRS/       I,          IBC,         IC,         ICOL,
     X       INEW,        INOCH,      IOLD,        IOUT,       IR,
     X       IRC,         IREP,       IS,          ITC,        J,
     X       K,           KBAR
C
C***********************************************************************
C
      EQUIVALENCE (NARAY,CORR), (S,BO), (SSQ,REPVAR), (ZCUT,SUMXX)
      DIMENSION NARAY(1830),S(9),SSQ(9)
      DIMENSION  ZOUT(1)
C***********************************************************************
C       ZERO OUT ALL DATA ARRAYS EACH NEW DATA SET
  100 IZOUT=9470
      DO 101 J=1,IZOUT
  101 SUMXX(J) = 0.0
C
C***********************************************************************
C     READ IDENTIFICATION CARD AND OPTIONS CARD
C
      READ(INPUT5,110) I,IDENT
      WRITE(LIST,111) IDENT
      FIRST=.TRUE.
      ERRFXD=.FALSE.
  113 IF(I) 120,120,115
  115 READ(INPUT5,300) FMT
      WRITE(LIST,301) FMT
      I=I-1
      GO TO 113
  120 READ(INPUT5,1282) NOVAR,NODEP,NOTERM,NOOB
      WRITE(LIST,1283) NOVAR,NODEP,NOTERM,NOOB
      READ(INPUT5,117)BZERO,IFTT,IFWT,IFCHI,STORYC,STORYX,STORYI, IFSSR
     X      ,ECONMY,ISTRAT
      WRITE(LIST,118) BZERO,IFTT,IFWT,IFCHI,STORYC,STORYX,STORYI, IFSSR
     X      ,ECONMY,ISTRAT
C
```

```
C*************************************************************************
C      THESE ARE INITIALIZATIONS MADE BEFORE EACH SET OF DATA
C  ICOL DETERMINES THE NUMBER OF VARIABLES READ PER OBSERVATION
C  JCOL IS THE NUMBER OF TERMS IN THE TOTAL REGRESSION EQUATION
C  LENGTH IS THE NUMBER OF STORES NEEDED FOR THE MATRICES
      LENGTH= NOTERM*(NOTERM+1)/2
      ICOL=NOVAR + NODEP
      JCOL = NOTERM +NODEP
      NWHERE= NOTERM
      REWIND INTER
      DO 140  J=1,60
      IDCUT(J) = J
  140 NTERM(J)=J
      DO 145 J=1,100
      NXCOD(J)=J
      NTRAN(J)=0
  145 NCON(2*J)=J
C
C*************************************************************************
      IF(BZERO) WRITE(LIST,190)
      IF(.NOT.BZERO) WRITE(LIST,170)
C*************************************************************************
      READ(INPUT5,282) NTRANS,KONNO
      IF(NTRANS.EQ.0) GO TO 255
  220 READ  (INPUT5,230)(NTERM(K),K=1,JCOL)
      WRITE(LIST,235) (NTERM(K),K=1,JCOL)
      READ  (INPUT5,230)(NXCOD(I),NTRAN(I),NCON(2*I-1),NCON(2*I),I=1,NTR
     AANS )
      WRITE(LIST,240) (NXCOD(I),NTRAN(I),NCON(2*I-1),NCON(2*I),I=1,
     X   NTRANS)
      IF(KONNO) 255,255,250
  250 READ  (INPUT5,260)(CON(I),I=1,KONNO)
      WRITE(LIST,262) (CON(I),I=1,KONNO)
C*************************************************************************
C
  255 READ(INPUT5,257) DELETE,P
      IF(DELETE) IFTT=.TRUE.
C
C*************************************************************************
C     IF THERE ARE REPLICATED POINTS READ IN THE NUMBER OF POINTS AND
C      THE NUMBER OF REPLICATIONS.  SINGLE DATA POINTS ARE DATA POINTS
C     REPLICATED ONCE.  OBSERVED DATA MUST BE ARRANGED IN THE ORDER
C     IMPLIED HERE.
      READ(INPUT5,257) REPS
      XSAVE=.FALSE.
  265 IF(.NOT.REPS) GO TO 290
      READ(INPUT5,282) IREP,(NARAY(I),I=1,IREP)
      NPDEG=0
      IREP=1
      IC=NARAY(1)
      XSAVE=.TRUE.
      DO 315 I=1,NODEP
      POOLED(I)=0.0
      S(I)=0.0
  315 SSC(I)=0.0
C
C*************************************************************************
C     READ VARIABLE FORMAT FOR DATA
  290 READ(INPUT5,110) INPUT,FMT
      WRITE(LIST,111) FMT
  310 TOTWT=0.0D0
      WEIGHT=1.0D0
       WRITE(LIST,301) IDENT
C
```

```
C*************************************************************************
C       READ IN INPUT VARIABLES
      DO 490 J=1,NOOB
  330 IF(.NOT.IFWT) GO TO 350
  340 READ (INPUT,FMT) (X(I),I=1,ICOL)
      GO TO 360
  350 READ (INPUT,FMT)(X(I),I=1,ICOL), WEIGHT
  360 CONTINUE
      IF(ECONMY) WRITE(LIST,381) J,(X(I),I=1,ICOL)
  381 FORMAT(1H I4,9G14.6/(5X,9G14.6))
      IF(NTRANS.EQ.0) GO TO 450
      IF(ECONMY) GO TO 390
      WRITE(LIST,370)WEIGHT,J
      WRITE (LIST,380)(X(I),I=1,ICOL)
  390 CALL TRANS
  420 DO 430        K=1,JCOL
      I=NTERM(K)
      X(K) = CON(I)
  430 CONTINUE
  450 CONTINUE
      IF(ECONMY) GO TO 4609
      WRITE(LIST,460) J
  461 WRITE (LIST,380)(X(I),I=1,JCOL)
 4609 CONTINUE
      IF(IFCHI) WRITE(INTER) (X(I),I=1,69),WEIGHT
      IF(.NOT.XSAVE) GO TO 4611
      DO 4610 K=1,NOTERM
 4610 XCHK(K)=X(K)
 4611 CONTINUE
C
C*************************************************************************
C       COMPUTE THE ERROR VARIANCE FROM REPLICATED DATA
      IF(.NOT.REPS) GO TO 480
      IGOTO =1
      IF(NARAY(IREP).GT.1) IGOTO=2
      IF(J.GE.IC) WRITE(6,462) IREP
      DO 475 I=1,NODEP
      IF(I-1) 4629,4629,464
 4629 DO 463 K=1,NOTERM
      IF(X(K).NE.XCHK(K)) GO TO 2001
  463 CONTINUE
  464 CONTINUE
      KBAR=NOTERM+I
      S(I)=S(I)+X(KBAR)
      SSQ(I)=SSQ(I)+X(KBAR)**2
      IF(J-IC) 475,465,465
  465 GO TO (468,466), IGOTO
  466 ZEAN(1)=S(I)/FLOAT(NARAY(IREP))
      SSQ(I) = SSQ(I) - ZEAN(1)*S(I)
      POOLED(I)=POOLED(I)+SSQ(I)
      WRITE(6,467) I,SSQ(I),S(I),ZEAN(1)
  468 IF(I.LT.NODEP) GO TO 469
      NPDEG=NPDEG+NARAY(IREP) -1
      IREP=IREP+1
      IC = IC + NARAY(IREP)
      WRITE(LIST,4671)
  469 S(I)=0.0
      SSQ(I)=0.0
      XSAVE=.TRUE.
  475 CONTINUE
C
```

```
C************************************************************************
C     CALCULATE SUMS, SUMS OF SQUARES AND SUMS OF CROSS PRODUCTS.
  480 CALL SUMUP
  490 CONTINUE
C     490 CONTINUE IS THE END OF THE LOOP FOR READING DATA CARDS
      IF(.NOT.REPS) GO TO 496
      DO 493 I=1,NODEP
      REPVAR(I)=POOLED(I)/FLOAT(NPDEG)
  493 CONTINUE
  496 CONTINUE
C
C************************************************************************
C     ALL DATA HAS BEEN READ IN AND THE   XTRANSPOSEX AND XTRANSPOSEY
C     MATRIX HAVE BEEN CALCULATED.
C     NOW WRITE THE MATRICES
      CALL MFIX
      REWIND INTER
      GO TO 640
C
C************************************************************************
C     THIS CODING DELETES THE DATA FROM THE CORRELATION MATRIX
C     CORRESPONDING TO THE INDEPENDENT TERM DELETED
 6500 CONTINUE
      IR=IOUT-1
      IC= NOTERM - IOUT
      IF ( IC.EQ. 0) GO TO 6700
      INOCH= IOUT*IR/2
      INEW = INOCH
      IOLD = INEW + IOUT
      IRC=0
      IBC=0
      ITC=0
      DO 6600 I=IOLD,LENGTH
      INEW = INEW+1
      IOLD=IOLD + 1
      IF(ITC.GT.0) GO TO 6540
      IRC=IRC + 1
      IF(IRC.GT.IR)  GO TO 6530
      CORR(INEW) = CORR(IOLD)
      GO TO 6600
 6530 IBC=IBC + 1
      ITC = IBC
      IOLD = IOLD+1
      IRC= 0
 6540 ITC = ITC -1
      CORR(INEW) = CORR(IOLD)
 6600 CONTINUE
 6700 LENGTH = LENGTH-NOTERM
      NOTERM= NOTERM -1
      JCOL= NOTERM+NODEP
      IF(BZERO) WRITE(LIST,670)
      IF(.NOT.BZERO) WRITE(LIST,560)
      IF(ECONMY) GO TO 640
      CALL TRIANG(CORR,NOTERM,8,FMTTRI)
C
C************************************************************************
C     INVERT THE CORR COEF MATRIX AND COMPUTE REGRESSION COEFS
C     AND SUMS OF SQUARES DUE TO REGRESSION IN THE MATRIX INVERSION
C     ROUTINE
  640 CONTINUE
      CALL MATINV
      FIRST=.FALSE.
C
```

```
C*********************************************************************
C     WRITE(XTX) INVERSE. THIS MATRIX TIMES ERROR MEAN SQUARE (ERRMS)
C     IS THE VARIANCE-COVARIANCE MATRIX OF REGRESSION COEFFICIENTS.
      IF(ECONMY) GO TO 970
      WRITE(LIST,700)
      CALL TRIANG(SUMXXI,NOTERM,8,FMTTRI)
C
C*********************************************************************
C     IF A VARIABLE HAS BEEN DELETED ADJUST COUNTERS AND RECOMPUTE THE
C     REGRESSION. IF NO VARIABLE HAS BEEN DELETED CONTROL WILL PASS
C     FROM THE TTEST ROUTINE TO THE CHI-SQUARE OPTION.
  970 CONTINUE
      IF(.NOT.IFTT) GO TO 1020
  980 WRITE (LIST,301) IDENT
      CALL TTEST($1020)
      IF(NODEP-1) 985,990,985
  985 WRITE(LIST,986) NODEP
      NODEP=1
  990 J=JCOL-1
      DO 995 K=IOUT,J
      NTERM(K)=NTERM(K+1)
      ZEAN(K) = ZEAN(K+1)
      SUMX(K) = SUMX(K+1)
      SUMX2(K) = SUMX2(K+1)
      IDOUT(K) = IDOUT(K+1)
      SUMXY(K,1)= SUMXY(K+1,1)
  995 CONTINUE
      IF(NOTERM.EQ.1) GO TO 1000
      GO TO 6500
 1000 WRITE(LIST,1005)
      NOTERM=0
      GO TO 1035
C
C*********************************************************************
C
 1020 IF(.NOT.IFCHI) GO TO 1035
 1030 WRITE(LIST,301) IDENT
       CALL CHISQ
C
C*********************************************************************
 1035 READ(INPUT5,117) PREDCT
      IF(.NOT.PREDCT) GO TO 100
      CALL PREDIC
 1040 GO TO 100
C
C*********************************************************************
 2001 WRITE(LIST,1806)
      STOP
C*********************************************************************
 8001 FORMAT(1H1)
 8002 FORMAT(1H2)
  110 FORMAT (I2,13A6)
  111 FORMAT (1H1,13A6,A2)
  117 FORMAT(9L1,I1)
  118 FORMAT(1H 9L1,I1)
  170 FORMAT(33H THERE IS NO B0 TERM IN THE MODEL)
  190 FORMAT(26H THERE IS A B0 TO ESTIMATE )
  230 FORMAT(40I2)
  235 FORMAT(11H NTERM(K)=  /(1H 30I4))
  240 FORMAT(25H THE TRANSFORMATIONS ARE  /(1H 5(4I4,5X)))
  257 FORMAT( IL1, F3.3)
  260 FORMAT(5E15.7)
  262 FORMAT(19H THE CONSTANTS ARE  /(01H 8G15.7)))
  282 FORMAT(20I4)
  283 FORMAT(1H 20I4)
```

```
  300 FORMAT(13A6,1A2)
  301 FORMAT (1H 13A6,A2)
  370 FORMAT(1H0,29HOBSERVED VARIABLES, WEIGHT = G14.6,6X,15HOBSERVATION
     1 = ,I5)
  380 FORMAT(1H  9G14.6)
  460 FORMAT(1H ,37HTERMS OF THE EQUATION, OBSERVATION = ,I5)
  462 FORMAT(18HK** REPLICATE SET  I5,3X,100(1H*))
 4671 FORMAT(1H 125(1H*))
  467 FORMAT(14H  DEP. VAR.    I6,8H    SSQ=G14.7,8H    SUM=G14.7,8H   M
     XEAN= G14.7)
  500 FORMAT(1H0,21HTOTAL OF THE WEIGHTS ,F8.0)
  510 FORMAT(1H0,69HTOTAL WEIGHT TOO HIGH, EXCEEDS NUMBER OF OBSERVATION
     1S BY ONE PERCENT. )
  520 FORMAT(1H0,69HTOTAL WEIGHT TOO LOW, LESS THAN 95 PERCENT OF NUMBER
     1 OF OBSERVATIONS. )
  540 FORMAT(1H 8G14.7)
  560 FORMAT(21H2X TRANSPOSE X MATRIX     )
  670 FORMAT(25H2CORRELATION COEFFICIENTS    )
  700 FORMAT(32H2(X TRANSPOSE X) INVERSE MATRIX        )
  986 FORMAT(39H THE NUMBER OF DEPENDENT VARIABLES WAS  I3,83H  IT IS BE
     XING SET TO ONE AND THE REJECTION OPTION EXERCISED ON DEPENDENT VAR
     XIABLE 1    )
 1005 FORMAT(39H THERE IS NO EVIDENCE OF A REGRESSION.      /
     X     74H USE THE MEAN RESPONSE FOR THE BEST ESTIMATE OF THE DEPEND
     XENT VARIABLE(S).     )
 1282 FORMAT(3I4,I5)
 1283 FORMAT(1H 3I4,I5)
 1306 FORMAT(40H REPLICATE SETS ARE NOT GROUPED PROPERLY    )
      END
```

```
$IBFTC TRANSX

      SUBROUTINE TRANS
C***********************************************************************
C
      COMMON/BIG/SUMXX(1830),SUMXXI(1830),EIG(1830),SUMXY(60,9),
     X  B(60,9),CORR(1830)
      COMMON/MED/       BO(9),              CON(99),            ERRMS(9),
     X   IDENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                            RESMS(9),           SUMX(138),
     XSUMX2(69),        X(99),              ZEAN(69),           SUMY2(18)
      COMMON/SMALL/     BYPASS,BZERO,DELETE, FIRST, IFCHI,   IFSSR,
     X       IFTT,      IFWT,        INPUT,       INPUT5,      INTER,
     X       ISTRAT,    JCOL,        KONNO,       LENGTH,      LIST,
     X       NERROR,    NODEP,       NCOB,                     NOTERM,
     X       NOVAR,     NPDEG,       NRES,        NTRANS,      NWHERE,
     X       P,         PREDCT,      REPS,        RWT,
     X       STORYI,     STORYC,      STORYX,      TOTWT,       WEIGHT,
     X       ERRFXD, ECONMY
      LOGICAL ECONMY
      LOGICAL      BYPASS,      BZERO,       DELETE,      IFCHI,
     XIFSSR,       IFTT,        IFWT,        REPS,        PREDCT,
     XSTORYC,      STORYX,      STORYI,      FIRST ,ERRFXD
```

```
      DOUBLE PRECISION RWT,TOTWT,WEIGHT
      COMMON/CNTRS/      I,           IBC,          IC,          ICOL,
     X       INEW,       INOCH,       IOLD,         IOUT,        IR,
     X       IRC,        IREP,        IS,           ITC,         J,
     X       K,          KBAR
C
C*********************************************************************
C     THIS SUBROUTINE PERFORMS TRANSFORMATIONS IF THIS OPTION IS
C        REQUESTED.
C
C
C     K                 TRANSFORMATION SET NUMBER.
C     NCON(2*K-1)        CONSTANT NUMBER TO USE.
C     NCON(2*K)          DERIVED CONSTANT.
C     NTRAN(K)           NUMBER OF TRANSFORMATION REQUESTED.
C     NXCOC(K)           VARIABLE NUMBER
C
   80 DO 5CO K=1,NTRANS
      I=NCON(2*K-1)
      IF(I)100,100,110
  100 CONS=0.0
      GO TC 120
  110 CONS=CON(I)
  120 I=NXCOD(K)
      Y=X(I)
      MTRAN = NTRAN(K)
      IF(MTRAN.LE.0) MTRAN*31
  140 GO TC(150,160,170,180,190,200,210,220,230,240,250,260,270,280,290,
     A300,310,320,330,340,350,360,370,380,390,400,410,420,430,440,450),
     B MTRAN
  150 CONS=Y+CONS
      GO TC 460
  160 CONS=Y*CONS
      GO TO 460
  170 CONS=CONS/Y
      GO TC 460
  180 CONS=EXP(Y)
      GO TC 460
  190 CONS=Y**CONS
      GO TC 460
  200 CCNS=ALOG(Y)
      GO TC 460
  210 CONS=ALOG10(Y)
      GO TC 460
  220 CONS=SIN(Y)
      GO TC 460
  230 CCNS=COS(Y)
      GO TO 460
  240 CONS=SIN(3.14159265*(CONS*Y) )
      GO TC 460
  250 CONS=COS(3.14159265*(CONS*Y) )
      GO TO 460
  260 CONS=1.0/Y
      GO TC 460
  270 CONS=EXP(CONS/Y)
      GO TO 460
  280 CONS=EXP(CONS/(Y*Y))
      GO TC 460
  290 CONS=SQRT(Y)
      GO TC 460
  300 CONS=1.0/SQRT(Y)
      GO TC 460
  310 CONS=CONS**Y
      GO TO 460
  320 CONS=10.0**Y
```

```
      GO TC 460
  330 CONS=SINH(Y)
      GO TO 460
  340 CONS=COSH(Y)
      GO TC 460
  350 CONS=(1.0-COS(Y))/2.0
      GO TO 460
  360 CONS=ATAN(Y)
      GO TC 460
  370 CONS=ATAN2(Y/CONS)
      GO TC 460
  380 CONS=Y*Y
      GO TC 460
  390 CONS=Y*Y*Y
      GO TO 460
  400 CONS=ARSIN(SQRT(Y))
      GO TC 460
  410 CONS=2.0*3.14159265*Y
      GO TO 460
  420 CONS=1.0/(2.0*3.14159265*Y)
      GO TO 460
  430 CONS=ERF(Y)
      GO TO 460
  440 CONS=GAMMA(Y)
      GO TO 460
  450 CONS=Y
  460 I=NCON(2*K)
      IF(I)470,470,480
  470 CON(K)=CONS
      GO TC 500
  480 CON(I)=CONS
      IF(I-60) 500,500,490
  490 X(I)=CONS
  500 CONTINUE
      RETURN
      END
```

$IBFTC SUMUPX

```
C
C     SUBROUTINE SUMUPS
C
C     PURPOSE
C         1)CALCULATE (X TRANSPOSE X) AND (X TRANSPOSE Y) MATRICES ONE
C           OBSERVATION AT A TIME.
C         2)COMPUTE TOTAL OF THE WEIGHTS
C         ** BOTH CALCULATIONS ARE IN DOUBLE PRECISION
C
C     SUBROUTINES NEEDED
C           LOC
C
C*************************************************************************
      SUBROUTINE SUMUP
      COMMON/BIG/SUMXX(1830),EIG(1830),SUMXY(60,9),CORR(1830)
      DOUBLE PRECISION SUMXX,SUMXY
      COMMON/MED/       BO(9),              CON(99),            ERRMS(9),
     X   IDENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                            RESMS(9),           SUMX(69),
     XSUMX2(69),        X(99),              ZEAN(69),           SUMY2(9)
      DOUBLE PRECISION SUMX,SUMY2
      COMMON/SMALL/      BYPASS,BZERO,DELETE, FIRST, IFCHI,     IFSSR,
     X      IFTT,       IFWT,        INPUT,      INPUT5,        INTER,
     X      ISTRAT,     JCOL,        KONNO,      LENGTH,        LIST,
     X      NERROR,     NODEP,       NOOB,                      NOTERM,
     X      NOVAR,      NPDEG,       NRES,       NTRANS,        NWHERE,
     X      P,          PREDCT,      REPS,       RWT,
     X      STORYI,      STORYC,      STORYX,     TOTWT,         WEIGHT,
     X      ERRFXD, ECONMY
      LOGICAL ECONMY
      LOGICAL     BYPASS,     BZERO,      DELETE,     IFCHI,
     XIFSSR,      IFTT,       IFWT,       REPS,       PREDCT,
     XSTORYC,     STORYX,     STORYI,     FIRST ,ERRFXD
      DOUBLE PRECISION RWT,TOTWT,WEIGHT
      COMMON/CNTRS/     I,          IBC,        IC,         ICOL,
     X      INEW,       INOCH,      IOLD,       IOUT,       IR,
     X      IRC,        IREP,       IS,         ITC,        J,
     X      K,          KBAR
C
C*************************************************************************
      DO 110 I=1,JCOL
      SUMX(I)=SUMX(I)+X(I)*WEIGHT
  110 CONTINUE
      DO 100 K=1,NOTERM
      DO 90 J=1,NODEP
      KBAR=J+NOTERM
      SUMXY(K,J) = SUMXY(K,J) + X(K)*X(KBAR)*WEIGHT
   90 CONTINUE
      DO 50 I=1,K
C
      CALL LOC(K,I,IR)
      SUMXX(IR) = SUMXX(IR) + X(I)*X(K)*WEIGHT
   50 CONTINUE
  100 CONTINUE
      DO 15 J=1,NODEP
      KBAR=NOTERM + J
   15 SUMY2(J)=SUMY2(J)+X(KBAR)**2*WEIGHT
      TOTWT=TOTWT+WEIGHT
      RETURN
      END
```

```
$IBFTC MFIXXX

      SUBRCUTINE MFIX
C     THIS ROUTINE USES THE SUMXX MATRIX COMPUTE IN DOUBLE PRECISION
C     AND THE SUMX ARRAY COMPUTED IN DOUBLE PRECISION TO COMPUTE THE
C     SUMXX DEVIATIONS FORM OF X TRANSPOSE X IN DOUBLE PRECISION AND
C     THE RESULT IS TRUNCATED TO SINGLE.
C     THE MATRICES ARE ALSO PRINTED
C
C*********************************************************************
      COMMON/BIG/SUMXX(1830),EIG(1830),SUMXY(60,9),CORR(1830)
      DOUBLE PRECISION  SUMXX,SUMXY
      COMMON/MED/         BO(9),              CON(99),          ERRMS(9),
     X   ICENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                              RESMS(9),         SUMX(69),
     XSUMX2(69),          X(30),              ZEAN(69),         SUMY2(9)
      DOUBLE PRECISION SUMX,SUMX2,SUMY2
       COMMON /FRMTS/ FMT(13),FMTTRI(14)
      COMMON/SMALL/      BYPASS,BZERO,DELETE, FIRST, IFCHI,  IFSSR,
     X       IFTT,       IFWT,       INPUT,      INPUT5,     INTER,
     X       ISTRAT,     JCOL,       KONNO,      LENGTH,     LIST,
     X       NERROR,     NODEP,      NOOB,                   NOTERM,
     X       NOVAR,      NPDEG,      NRES,       NTRANS,     NWHERE,
     X       P,          PREDCT,     REPS,       RWT,
     X       STORYI,      STORYC,     STORYX,      TOTWT,      WEIGHT,
     X       ERRFXD, ECONMY
      LOGICAL ECONMY
      DOUBLE PRECISION RWT,TOTWT,WEIGHT
      LOGICAL      BYPASS,     BZERO,      DELETE,     IFCHI,
     XIFSSR,       IFTT,       IFWT,       REPS,       PREDCT,
     XSTORYC,      STORYX,     STORYI,     FIRST ,ERRFXD
      COMMON/CNTRS/      I,          IBC,        IC,         ICOL,
     X       INEW,       INOCH,      IOLD,       IOUT,       IR,
     X       IRC,        IREP,       IS,         ITC,        J,
     X       K,          KBAR
      EQUIVALENCE (EIG,SNGLX),(EIG(70),RITEXY),(SNGLXX,SUMXX),(SNGLXY,
     X  SUMXY),(SNGLX2,SUMX2) ,(SNGLY2,SUMY2)
      DIMENSION SNGLX(69),SNGLXY(60,9),SNGLXX(1830) ,RITEXY(60,9)
     X    ,SNGLX2(69) ,SNGLY2(9)
C*********************************************************************
C
      DO 10 J=1,JCOL
   10 SNGLX(J)= SNGL(SUMX(J))
      IF(ECCNMY) GO TO 500
      WRITE(LIST,530)
      WRITE (LIST,540) (SNGLX(I),I=1,JCOL)
      WRITE(LIST,560)
      DO 20 I=1,LENGTH
   20 CORR(I)= SNGL(SUMXX(I))
      CALL TRIANG(CORR,NOTERM,8,FMTTRI)
      DO 30 I=1,NOTERM
      DO 30 J=1,NODEP
   30 RITEXY(I,J)= SNGL(SUMXY(I,J))
      WRITE(LIST,565)
      CALL RECT(NOTERM,NODEP,60,9,RITEXY,FMTTRI)
C
C*********************************************************************
C     COMPUTE AND PRINT MEANS. COMPUTE AND PRINT THE(X TRANSPOSE X)
C     MATRIX IN TERMS OF DEVIATIONS FROM MEAN. THE DEVIATIONS FORM
C     OF (X T X)  IS THE VARIANCE-COVARIANCE MATRIX OF THE
C     INDEPENDENT VARIABLES.
```

```
  500 CONTINUE
      RWT=1.0D0/TOTWT
      DO 570 I=1,JCOL
  570 ZEAN(I)=SUMX(I)*RWT
      WRITE(LIST,580)
      WRITE(LIST,540)  (ZEAN(I),I=1,JCOL)
      IR = 0
      DO 600 J=1,NOTERM
      IR=IR + J
      IF(.NOT.BZERO) GO TO 601
      SUMX2(J)=SUMXX(IR)-SUMX(J)**2 *RWT
      GO TC 600
  601 SUMX2(J)=SUMXX(IR)
  600 CONTINUE
  602 CONTINUE
      IR=1
      DO 620 J=1,NOTERM
      DO 618 K=1,NODEP
      IF(BZERO) GO TO 617
      SNGLXY(J,K)=SNGL(SUMXY(J,K))
      GO TO 618
  617 KBAR=NOTERM+K
      SNGLXY(J,K)=SNGL(SUMXY(J,K)-SUMX(J)*SUMX(KBAR)*RWT)
  618 CONTINUE
  619 DO 620 K=1,J
      IF(.NCT.BZERO) GO TO 6191
      SUMXX(IR)=SUMXX(IR)-SUMX(K)*SUMX(J)*RWT
 6191 CORR(IR)=SNGL(SUMXX(IR)/DSQRT(SUMX2(J)*SUMX2(K)))
 6193 SNGLXX(IR)=SNGL(SUMXX(IR))
 6194 IR=IR+1
  620 CONTINUE
C***********************************************************************
      IF(ECONMY) GO TO 622
      IF(.NOT.BZERO) GO TO 621
      WRITE(LIST,625)
      CALL TRIANG(SNGLXX,NOTERM,8,FMTTRI)
      WRITE(LIST,630)
      CALL RECT(NOTERM,NODEP,60,9,SNGLXY,FMTTRI)
  621 WRITE(LIST,670)
      CALL TRIANG(CORR,NOTERM,8,FMTTRI)
 622  CONTINUE
      DO 640 J=1,NOTERM
  640 SNGLX2(J)=SNGL(SUMX2(J))
      DO 650 J=1,NODEP
      IF(BZERO) GO TO 645
      SNGLY2(J)=SNGL(SUMY2(J))
      GO TO 650
  645 K=NCTERM+J
      SNGLY2(J)=SNGL(SUMY2(J)-SUMX(K)**2*RWT)
  650 CONTINUE
C***********************************************************************
      RETURN
C
  530 FORMAT(1H0,32H SUMS OF INDEP AND DEP VARIABLES )
  540 FORMAT(1H  8G15.7)
  560 FORMAT(21H2X TRANSPOSE X MATRIX     )
  565 FORMAT(21H2X TRANSPOSE Y MATRIX     )
  580 FORMAT(33H MEANS OF INDEP AND DEP VARIABLES     )
  625 FORMAT(53H2X TRANSPOSE X MATRIX WHERE X IS DEVIATION FROM MEAN  )
  630 FORMAT(60H2X TRANSPOSE Y MATRIX WHERE X AND Y ARE DEVIATIONS FROM
     XMEAN  )
  670 FORMAT(25H2CORRELATION COEFFICIENTS    )
      ENC
```

```
$IBFTC MATINX

C
C     SUBROUTINE MATINV
C
C***********************************************************************
C
C     PURPOSE
C         1)COMPUTE EIGENVALUES AND EIGENVECTORS OF (X-TRANSPOSE X)
C           AND/OR CORRELATION MATRIX IF REQUESTED.
C           STORYC =.TRUE. IF FOR CORRELATION
C           STORYX = .TRUE IF FOR XTX
C         2)INVERT CORRELATION COEFFICIENT MATRIX BY EITHER BORDERING
C           OR GAUSS ELIMINATION
C         3)COMPUTE PRODUCT OF CORRELATION AND INVERTED CORRELATION
C           MATRIX IF REQUESTED
C           STORYI = .TRUE. IF THE PRODUCT IS TO BE PRINTED
C         4)COMPUTE (X TRANSPOSE X) INVERSE FROM INVERTED CORRELATION
C           MATRIX
C         5)COMPUTE REGRESSION COEFFICIENTS
C         6)COMPUTE OTHER REGRESSION STATISTICS
C
C     SUBRCUTINES NEEDED
C           BORD
C           LOC
C           EIGEN
C           INVXTX
C           RECT
C           RSTATS
C           TRIANG
C
C     REMARKS
C           THE EIGENVALUES ARE COMPUTED AS AN AID IN DETERMINING THE
C           CONDITION OF THE SYSTEM OF EQUATIONS FOR THE REGRESSION
C           COEFFICIENTS. EXAMINATION OF THEM AND THEIR ASSOCIATED
C           EIGENVECTORS MAY SHOW THAT CERTAIN SETS OF INDEPENDENT
C           VARIABLES ARE HIGHLY CORRELATED AND NOT EASILY LIABLE TO
C           INDEPENDENT STUDY.
C
      SUBROUTINE MATINV
C
C***********************************************************************
      COMMON/BIG/SUMXX(1830),SUMXXI(1830),EIG(1830),SUMXY(60,9),
     X  B(60,9),CORR(1830)
      COMMON/MED/       BO(9),              CON(99),             ERRMS(9),
     X   IDENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                            RESMS(9),            SUMX(138),
     XSUMX2(69),        X(99),              ZEAN(69),            SUMY2(18)
       COMMON /FRMTS/ FMT(13),FMTTRI(14)
      COMMON/SMALL/     BYPASS,BZERO,DELETE, FIRST, IFCHI,  IFSSR,
     X       IFTT,        IFWT,         INPUT,       INPUT5,      INTER,
```

```
     X      ISTRAT,     JCOL,       KONNO,      LENGTH,     LIST,
     X      NERROR,     NODEP,      NOOB,                   NOTERM,
     X      NOVAR,      NPDEG,      NRES,       NTRANS,     NWHERE,
     X      P,          PREDCT,     REPS,       RWT,
     X      STORYI,      STORYC,     STORYX,     TOTWT,      WEIGHT,
     X      ERRFXD, ECONMY
      LOGICAL ECONMY
      LOGICAL      BYPASS,     BZERO,      DELETE,     IFCHI,
     XIFSSR,      IFTT,       IFWT,       REPS,       PREDCT,
     XSTORYC,     STORYX,     STORYI,     FIRST ,ERRFXD
      DOUBLE PRECISION RWT,TOTWT,WEIGHT
      COMMON/CNTRS/     I,          IBC,        IC,         ICOL,
     X      INEW,       INOCH,      IOLD,       IOUT,       IR,
     X      IRC,        IREP,       IS,         ITC,        J,
     X      K,          KBAR
      DIMENSION A(1),C(1),XTX(3),CMAT(3),HOL(3)
      EQUIVALENCE  (SUMXX,A),(SUMXXI,C)
      DATA  (XTX(I),I=1,3) /6HX TRAN, 6HSPOSE , 6HX     /
      DATA  (CMAT(I),I=1,3)/ 6HCORREL , 6HATION  ,1H /
C
C*********************************************************************
      IORDER= NOTERM
      IF(NOTERM-1) 10,10,12
   10 SUMXXI(1)= 1.0/SUMX2(1)
      GO TO 350
C
C      TRANSFER CORR TO A FOR INVERSION
C     AT THIS POINT THE INFORMATION IN A MAY BE USED FOR ANY DESIRED
C     CALCULATIONS --- EIGENVALUES,RANK,ETC.
C     JUST PUT CORR INTO A BEFORE PROCEEDING TO REMAINDER OF ROUTINE
C
   12 BYPASS=.FALSE.
      IF(.NOT. STORYX) GO TO 30
   15 DO 14  I=1,3
   14 HOL(I)=XTX(I)
   16 CALL EIGEN(A,SUMXXI,IORDER,0)
      WRITE(LIST,17)(HOL(I),I=1,3)
      J=0
      DO 18 I=1,IORDER
      J=J+I
   18 A(I)=A(J)
      WRITE(LIST,19) (A(I),I=1,IORDER)
      WRITE(LIST,20)
      CALL RECT(IORDER,IORDER,IORDER,IORDER,SUMXXI,FMTTRI)
   30 DO 35 I=1,LENGTH
   35 A(I)=CORR(I)
      IF(BYPASS)GO TO 49
      IF(.NOT.STORYC) GO TO 49
      DO 36 I=1,3
   36 HOL(I)= CMAT(I)
      BYPASS=.TRUE.
      GO TO 16
C
C*********************************************************************
C      NO SUBMODELS TO ANALYZE SO INVERT A DIRECTLY BY GAUSS
   49 IF(IFSSR) GO TO 50
```

```
      CALL INVXTX(A,NOTERM,D,1.0)
      GO TO 60
C
C*********************************************************************
CC      SUBMODELS HAVE BEEN REQUESTED SO WE USE BORDERING
   50 IORDER=0
   55 IORDER=IORDER +1
       CALL BORD(IORDER,A)
   60 IF(.NOT.STORYI) GO TO 200
C
C*********************************************************************
C     WRITE INVERSE OF CORRELATION COEFFICIENT MATRIX
      WRITE(LIST,65)
      CALL TRIANG(A,IORDER,8,FMTTRI)
C     COMPUTE A TIMES A INVERSE AND WRITE IT
      WRITE(LIST,70)
      ITC= 0
      DO 150 IC=1,IORDER
      DO 150 IR=1,IC
      ITC= ITC+1
      C(ITC)= 0.0
      DO 130 I=1,IORDER
      CALL LOC(IR,I,IRC)
      CALL LOC(I,IC,IBC)
      C(ITC)= C(ITC) + A(IRC)*CORR(IBC)
  130 CONTINUE
  150 CONTINUE
C
      CALL TRIANG(C,IORDER,8,FMTTRI)
C
  200 CONTINUE
C
C*********************************************************************
C     COMPUTE SUMXXI FROM CORR INVERSE.  SUMXXI TIMES THE ERROR MEAN
C     SQUARE IS THE VARIANCE-COVARIANCE MATRIX OF THE REGRESSION
C     COEFFICIENTS.
      IR=0
      DO 340 I=1,IORDER
      DO 340 J=1,I
      IR= IR+1
      SUMXXI(IR) = A(IR)/SQRT(SUMX2(I)*SUMX2(J))
  340 CONTINUE
C
C*********************************************************************
C     COMPUTE COEFFICIENTS AND PRINT THEM
C
  350 DO 370 J=1,NODEP
      DO 370 K=1,IORDER
      B(K,J)=0.0
      DO 370 L=1,IORDER
      CALL LOC(L,K,IR)
      B(K,J) = B(K,J) + SUMXXI(IR)*SUMXY(L,J)
  370 CONTINUE
C
      WRITE(LIST,380) IDENT
      WRITE(LIST,382)
```

```
      IF(.NOT.BZERO) GO TO 400
      DO 390 J=1,NODEP
      SUM=0.0
      KBAR= NOTERM + J
      DO 385 K=1,IORDER
      SUM = SUM + B(K,J)*ZEAN(K)
  385 CONTINUE
      BO(J)= ZEAN(KBAR) -SUM
  390 CONTINUE
      WRITE(LIST,395)
      WRITE(LIST,397) (BO(K),K=1,NODEP)
  400 WRITE(LIST,410)
      DO 430 J=1,IORDER
      WRITE(LIST,432) IDOUT(J),(B(J,K),K=1,NODEP)
  430 CONTINUE
C
C************************************************************************
C     COMPUTE REGRESSION STATISTICS IN RSTATS
C
      CALL RSTATS(IORDER)
C
C************************************************************************
C     IF IORDER IS LESS THAN NOTERM WE HAVE USED THE BORDERING OPTION
C     AND MUST GO BACK TO FINISH.
C
      IF(ICRDER-NOTERM) 55,500,500
  500 STORYC=.FALSE.
      STORYX=.FALSE.
      RETURN
   17 FORMAT(34H2THE FOLLOWING ARE EIGENVALUES OF  2A6,A1,  7H MATRIX)
   19 FORMAT(1H 8G16.7)
   20 FORMAT(132H2THIS IS THE MODAL MATRIX OR MATRIX OF EIGENVECTORS. EI
     1GENVECTORS ARE WRITTEN IN COLUMNS LEFT TO RIGHT IN SAME ORDER AS
     2EIGENVALUES )
   65 FORMAT(14H2CORR INVERSE )
   70 FORMAT(118H2AS A PARTIAL CHECK ON INVERSION ACCURACY THE (CORR)*(C
     XORR INVERSE) MATRIX FOLLOWS. IT SHOULD BE THE IDENTITY MATRIX.   )
  380 FORMAT(1H1,13A6,1A2)
  382 FORMAT( 61H EACH COLUMN CONTAINS THE COEFFICIENTS FOR ONE DEPENDEN
    XT TERM )
  395 FORMAT(20H CONSTANT TERM (BO) )
  397 FORMAT(4X,9G14.6)
  410 FORMAT(36H REGRESSION COEFFICIENTS (B1,....,BK) )
  432 FORMAT(1H I3,9G14.6)
      END
```

```
$IBFTC RSTATX
C
C     SUBROUTINE RSTATS
C
C     PURPOSE
C         1)COMPUTE AND PRINT THE ANALYSIS OF VARIANCE TABLES ON
C           REGRESSION AND LACK-OF-FIT IF APPROPRIATE.
C         2)COMPUTE AND PRINT R-SQUARED AND STANDARD ERROR OF
C           ESTIMATE
C         3)COMPUTE AND PRINT SUMS OF SQUARES DUE TO EACH VARIABLE
C           IF IT WERE LAST TO ENTER REGRESSION
C         4)COMPUTE AND PRINT THE STANDARD DEVIATIONS OF EACH
C           REGRESSION COEFFICIENT.
C
      SUBROUTINE RSTATS(IORDER)
C******************************************
      COMMON/BIG/SUMXX(1830),SUMXXI(1830),EIG(1830),SUMXY(60,9),
     X B(60,9),CORR(1830)
      COMMON/MED/     BO(9),           CON(99),          ERRMS(9),
     X   IDENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                       RESMS(9),         SUMX(138),
     XSUMX2(69),         X(99),        ZEAN(69),         SUMY2(18)
       COMMCN /FRMTS/ FMT(13),FMTTRI(14)
      COMMON/SMALL/     BYPASS,BZERO,DELETE, FIRST, IFCHI,  IFSSR,
     X      IFTT,       IFWT,        INPUT,      INPUT5,      INTER,
     X      ISTRAT,     JCOL,        KONNO,      LENGTH,      LIST,
     X      NERROR,     NODEP,       NOOB,                    NOTERM,
     X      NOVAR,      NPDEG,       NRES,       NTRANS,      NWHERE,
     X      P,          PREDCT,      REPS,       RWT,
     X      STORYI,       STORYC,      STORYX,     TOTWT,       WEIGHT,
     X      ERRFXD, ECONMY
      LOGICAL ECONMY
      LOGICAL SATRTD
      LOGICAL      BYPASS,     BZERO,      DELETE,     IFCHI,
     XIFSSR,       IFTT,       IFWT,       REPS,       PREDCT,
     XSTORYC,      STORYX,     STORYI,     FIRST ,ERRFXD
      DOUBLE PRECISION RWT,TOTWT,WEIGHT
      COMMON/CNTRS/     I,           IBC,        IC,          ICOL,
     X      INEW,       INOCH,       IOLD,       IOUT,        IR,
     X      IRC,        IREP,        IS,         ITC,         J,
     X      K,          KBAR
C
C*****************************************************************
C      PROGRAMMING NOTE***********
C      SOME OF THESE EQUIVALENCES ARE USED TO COMMUNICATE WITH
C      SUBROUTINE PREDCT.  BE CAREFUL ABOUT CHANGES INVOLVING THE
C      ARRAY EIG .
      DIMENSION SSQREG(9),   SSQRES(9),    REGMS(9),
     X      XLOF(9),    XLOFMS(9),   FRATIO(9),  RSQD(9),     R(9),
     X      SSQLST(9),  DEVB(60,9)
```

```
      EQUIVALENCE (EIG(1),SSQREG),          (EIG(10),SSQRES),
     X            (EIG(19),REGMS),          (EIG(37),XLOF),
     X            (EIG(46),XLOFMS),         (EIG(55),FRATIO),
     X            (EIG(64),RSQD),           (EIG(73),R),
     X            (EIG(82),SSQLST),         (EIG(91),DEVB)
C
C***********************************************************************
C     COMPUTE DEGREES OF FREEDOM AND RECIPROCALS
      NREG= IORDER
      NTOT= IFIX(TOTWT)-1
      IF(.NOT.BZERO) NTOT= NTOT+1
      NRES= NTOT-NREG
      NLOF= NRES - NPDEG
      RNREG= 1.0/FLOAT(NREG)
      IF(NRES.EQ.0) GO TO 980
      RNRES= 1.0/FLOAT(NRES)
      SATRTD=.FALSE.
      IF(NLOF.EQ.0) GO TO 90
      RNLOF=1.0/FLOAT(NLOF)
      GO TO 100
   90 SATRTD=.TRUE.
  100 CONTINUE
      NXTERM=IORDER
      RNOOB=RWT
C
C***********************************************************************
C      COMPUTE RESIDUAL SUM OF SQUARES, RESIDUAL VARIANCE, VARIANCE
C      FROM REPLICATIONS IF APPROPRIATE, AND THE F-RATIO OF MEAN SQUARE
C     LACK-OF-FIT AND MEAN SQUARE RESIDUALS.
      DO 210 J=1,NODEP
      SSQREG(J)=0.0
      DO 200 I=1,NXTERM
      SSQREG(J)=SSQREG(J) + B(I,J)*SUMXY(I,J)
  200 CONTINUE
      SSQRES(J)=SUMY2(J)-SSQREG(J)
      REGMS(J)= SSQREG(J)* RNREG
      RESMS(J)= SSQRES(J)*RNRES
      RSQD(J)=SSQREG(J)/SUMY2(J)
      R(J)=SQRT(RSQD(J))
      IF((.NOT.REPS).OR.SATRTD) GO TO 210
      XLOF(J)=SSQRES(J)-POOLED(J)
      XLOFMS(J)= XLOF(J)*RNLOF
      FRATIO(J)=XLOFMS(J)/REPVAR(J)
  210 CONTINUE
C
C***********************************************************************
C      DETERMINE WHICH ESTIMATE OF SIGMA SQUARED SHOULD BE USED IN
C      HYPOTHESIS TESTS.  PUT THE PROPER ONE IN ERRMS AND SET ERRFXD
C      TO TRUE IF THE PRESENT VALUE IS TO BE USED FOR ALL FOLLOWING
C      TESTS AND T-STATISTICS.
      IOUT=ISTRAT
      IF(ERRFXD) GO TO 250
      IF(ISTRAT.NE.3) GO TO 214
  211 DO 213 J=1,NODEP
  213 ERRMS(J)= RESMS(J)
      NERROR = NRES
```

```
      IOUT=3
      GO TO 250
  214 IF(ISTRAT.NE.1) GO TO 218
      IF(.NOT.REPS) GO TO 211
      DO 215 J=1,NODEP
  215 ERRMS(J)= REPVAR(J)
      NERROR= NPDEG
      ERRFXD= .TRUE.
      IOUT=1
      GO TO 250
  218 IF(FIRST.AND.(IORDER.EQ.NOTERM)) GO TO 220
      GO TO 211
  220 ERRFXD= .TRUE.
      DO 222 J=1,NODEP
  222 ERRMS(J)= RESMS(J)
      NERROR= NRES
      IOUT=2
C
C***********************************************************************
C     WRITE ANOVA TABLES
  250 IS=2
      IF(REPS) IS=4
      DO 500 J=1,NODEP
      IF(ERRMS(J).EQ.0.0) ERRMS(J)=1.0E-30
      WRITE(LIST,1001) IS,J
      WRITE(LIST,1002)
      WRITE(LIST,1003) SSQREG(J), NREG, REGMS(J)
      WRITE(LIST,1004) SSQRES(J), NRES, RESMS(J)
      WRITE(LIST,1005)
      WRITE(LIST,1006) SUMY2(J),NTOT
      WRITE(LIST,1007)
      WRITE(LIST,1500) RSQD(J), R(J)
      STD=SQRT(RESMS(J))
      WRITE(LIST,1600) STD
      WRITE(LIST,1700) IOUT,ERRMS(J),NERROR
      F=REGMS(J)/ERRMS(J)
      WRITE(LIST,1750)F,NREG,NERROR
      IF((.NOT.REPS).OR.SATRTD) GO TO 500
      WRITE(LIST,2001)
      WRITE(LIST,1002)
      WRITE(LIST,2005) XLOF(J), NLOF, XLOFMS(J)
      WRITE(LIST,2006) POOLED(J), NPDEG, REPVAR(J)
       WRITE(LIST,1004) SSQRES(J),NRES,RESMS(J)
      WRITE(LIST,1005)
      WRITE(LIST,2008) FRATIO(J)
      WRITE(LIST,1007)
  500 CONTINUE
C
C***********************************************************************
C     COMPUTE CONTRIBUTION OF EACH INDEPENDENT VARIABLE TO REG SUM
C     OF SQUARES AS IF IT WERE LAST TO ENTER
      WRITE(LIST,370)
      IR= 0
      DO 8635 K=1,NXTERM
      IR= IR+K
      DO 8632 J=1,NODEP
```

```
 8632 SSQLST(J)= B(K,J)**2/SUMXXI(IR)
      WRITE(LIST,380) IDOUT(K),(SSQLST(J),J=1,NODEP)
 8635 CONTINUE
C
C*********************************************************************
C     COMPUTE STANDARD DEVIATION OF REGRESSION COEFFICIENTS
      WRITE(LIST,375)
      IF(.NOT.BZERO) GO TO 959
      DO 910 J=1,NXTERM
      R(J)=0.0
      DO 910 I=1,NXTERM
      CALL LOC(I,J,IR)
      R(J)=R(J)+ZEAN(I)*SUMXXI(IR)
  910 CONTINUE
      XXT=0.0
      DO 920 J=1,NXTERM
  920 XXT=XXT+ZEAN(J)*R(J)
      DO 930 K=1,NODEP
  930 DEVB(1,K)=SQRT(ERRMS(K)*(RNOOB+XXT))
      K=0
      WRITE(LIST,380) K,(DEVB(1,J),J=1,NODEP)
  959 IR=0
      DO 970 J=1,NXTERM
      IR= IR+J
      DO 960 K=1,NODEP
      DEVB(J,K) =SQRT(ERRMS(K)*SUMXXI(IR))
  960 CONTINUE
       WRITE(LIST,380) IDOUT(J),(DEVB(J,KR),KR=1,NODEP)
  970 CONTINUE
C
C*********************************************************************
C     FORMATS
 1001 FORMAT(I1,42H ANOVA OF REGRESSION ON DEPENDENT VARIABLE I5)
 1002 FORMAT(1H 79(1H*)/79H       SOURCE          SUMS OF SQUARES     DEG
     XREES OF FREEDOM  MEAN SQUARES           /1H 79(1H-))
 1003 FORMAT(17H     REGRESSION      G20.8, 5X,I10,5X,G20.8)
 1004 FORMAT(17H     RESIDUAL        G20.8, 5X,I10,5X,G20.8)
 1005 FORMAT(1H 79(1H-))
 1006 FORMAT(17H     TOTAL           G20.8, 5X,I10)
 1007 FORMAT(1H 79(1H*))
 2001 FORMAT(1X/1X/22H  ANOVA OF LACK OF FIT   )
 2005 FORMAT(17H     LACK OF FIT     G20.8, 5X,I10,5X,G20.8)
 2006 FORMAT(17H     REPLICATION     G20.8, 5X,I10,5X,G20.8)
 2008 FORMAT(28H     F = MS(LOF)/MS(REPS) =  F10.3 )
  370 FORMAT(74H1 SUMS OF SQUARES DUE TO EACH VARIABLE IF IT WERE LAST T
     XO ENTER REGRESSION )
  375 FORMAT(115H2 STANDARD DEVIATION OF REGRESSION COEFFICIENTS (DERIVE
     XD FROM DIAGONAL ELEMENTS OF  (X TRANSPOSE X)INVERSE MATRIX ))
  380 FORMAT(1H I3,9G14.6)
 1500 FORMAT(40H       R SQUARED = SSQ(REG) / SSQ(TOT) =    F8.6,
     X 5X, 4HR = F7.6)
 1600 FORMAT(34H       STANDARD ERROR OF ESTIMATE   G14.6)
 1700 FORMAT(24H USING POOLING STRATEGY I2,25H THE ERROR MEAN SQUARE =
     X G14.7, 26H WITH DEGREES OF FREEDOM = I6)
 1750 FORMAT(5X,19HF=MS(REG)/MS(ERR)= F6.2,5X,13HCOMPARE TO F(I2,1H,I3,1
     XH))
      RETURN
  980 WRITE(LIST,981)
  981 FORMAT(41H ZERO RESIDUAL DEGREES OF FREEDOM. STOP.    )
      STOP
      END
```

```
$IBFTC TTEST
C***********************************************************************
C
C     SUBROUTINE TTEST
C
C     PURPCSE
C           COMPUTE THE T-STATISTICS FOR EACH REGRESSION TERM AND
C           ITS TWO TAILED SIGNIFICANCE LEVEL.  THEN DETERMINE THE
C           TERM WITH THE LEAST SIGNIFICANCE AND RETURN THIS
C           INFORMATION TO RAPIER.
C
C***********************************************************************
      SUBROUTINE TTEST(*)
C
C***********************************************************************
      COMMON/BIG/SUMXX(1830),SUMXXI(1830),EIG(1830),SUMXY(60,9),
     X  B(60,9),CORR(1830)
      COMMON/MED/       BO(9),          CON(99),           ERRMS(9),
     X   IDENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                        RESMS(9),          SUMX(138),
     XSUMX2(69),        X(99),          ZEAN(69),          SUMY2(18)
      COMMON/SMALL/     BYPASS,BZERO,DELETE, FIRST, IFCHI,  IFSSR,
     X      IFTT,       IFWT,       INPUT,      INPUT5,     INTER,
     X      ISTRAT,     JCOL,       KONNO,      LENGTH,     LIST,
     X      NERROR,     NODEP,      NOOB,                   NOTERM,
     X      NOVAR,      NPDEG,      NRES,       NTRANS,     NWHERE,
     X      P,          PREDCT,     REPS,       RWT,
     X      STORYI,      STORYC,     STORYX,     TOTWT,      WEIGHT,
     X      ERRFXD, ECONMY
      LOGICAL ECONMY
      LOGICAL      BYPASS,     BZERO,      DELETE,     IFCHI,
     XIFSSR,       IFTT,       IFWT,       REPS,       PREDCT,
     XSTORYC,      STORYX,     STORYI,     FIRST ,ERRFXD
      DOUBLE PRECISION WEIGHT,RWT
      COMMON/CNTRS/     I,          IBC,        IC,         ICOL,
     X      INEW,       INOCH,      IOLD,       IOUT,       IR,
     X      IRC,        IREP,       IS,         ITC,        J,
     X      K,          KBAR
C
C***********************************************************************
      LOGICAL MAKENU,NOZERO
      DIMENSION   T(35,13),PLEVEL(13)
      DIMENSION DEVB(60,9)     , PROB(60,9)      ,TT(60,9)
      EQUIVALENCE (EIG(91),DEVB,TT),      (EIG(650),PROB)
      EQUIVALENCE  (P,PWANT)
C
C***********************************************************************
      DATA  (PLEVEL(JJ),JJ=1,13) /0.10,0.20,0.30,0.40,0.50,0.60,0.70,
     10.80,0.90,0.95,0.98,0.99,0.999 /
      DATA    (T(1,JJ),JJ=1,13) /0.158,0.325,0.510,0.727,1.000,1.376,
```

```
     1 1.963,3.078,6.314,12.706,31.821,63.657,636.619  /,
     2   (T(2,JJ),JJ=1,13) /0.142,0.289,0.445,0.617,0.816,1.061,
     3 1.386,1.886,2.920,4.3027,6.965,9.925,31.598     /,
     4   (T(3,JJ),JJ=1,13) /0.137,0.277,0.424,0.584,0.765,0.978,
     5 1.250,1.638,2.353,3.1825,4.541,5.841,12.924     /,
     6   (T(4,JJ),JJ=1,13) /0.134,0.271,0.414,0.569,0.741,0.941,
     7 1.190,1.533,2.132,2.7764,3.747,4.604,8.610      /,
     8   (T(5,JJ),JJ=1,13) /0.132,0.267,0.408,0.559,0.727,0.920,
     9 1.156,1.476,2.015,2.5706,3.365,4.032,6.869      /,
     A   (T(6,JJ),JJ=1,13) /0.131,0.265,0.404,0.553,0.718,0.906,
     B 1.134,1.440,1.943,2.4469,3.143,3.707,5.959      /,
     C   (T(7,JJ),JJ=1,13) /0.130,0.263,0.402,0.549,0.711,0.896,
     D 1.119,1.415,1.895,2.3646,2.998,3.499,5.408      /,
     E   (T(8,JJ),JJ=1,13) /0.130,0.262,0.399,0.546,0.706,0.889,
     F 1.108,1.397,1.860,2.3060,2.896,3.355,5.041      /,
     G   (T(9,JJ),JJ=1,13) /0.129,0.261,0.398,0.543,0.703,0.883,
     H 1.100,1.383,1.833,2.2622,2.821,3.250,4.781      /,
     I  (T(10,JJ),JJ=1,13) /0.129,0.260,0.397,0.542,0.700,0.879,
     J 1.093,1.372,1.812,2.2281,2.764,3.169,4.587      /
  DATA  (T(11,JJ),JJ=1,13) /0.129,0.260,0.396,0.540,0.697,0.876,
     1 1.088,1.363,1.796,2.2010,2.718,3.106,4.437      /,
     2  (T(12,JJ),JJ=1,13) /0.128,0.259,0.395,0.539,0.695,0.873,
     3 1.083,1.356,1.782,2.1788,2.681,3.055,4.318      /,
     4  (T(13,JJ),JJ=1,13) /0.128,0.259,0.394,0.538,0.694,0.870,
     5 1.079,1.350,1.771,2.1604,2.650,3.012,4.221      /,
     6  (T(14,JJ),JJ=1,13) /0.128,0.258,0.393,0.537,0.692,0.868,
     7 1.076,1.345,1.761,2.1448,2.624,2.977,4.140      /,
     8  (T(15,JJ),JJ=1,13) /0.128,0.258,0.393,0.536,0.691,0.866,
     9 1.074,1.341,1.753,2.1315,2.602,2.947,4.073      /,
     A  (T(16,JJ),JJ=1,13) /0.128,0.258,0.392,0.535,0.690,0.865,
     B 1.071,1.377,1.746,2.1199,2.583,2.921,4.015      /,
     C  (T(17,JJ),JJ=1,13) /0.128,0.257,0.392,0.534,0.689,0.863,
     D 1.069,1.333,1.740,2.1098,2.567,2.898,3.965      /,
     E  (T(18,JJ),JJ=1,13) /0.127,0.257,0.392,0.534,0.688,0.862,
     F 1.067,1.330,1.734,2.1009,2.552,2.878,3.922      /,
     G  (T(19,JJ),JJ=1,13) /0.127,0.257,0.391,0.533,0.688,0.861,
     H 1.066,1.328,1.729,2.0930,2.539,2.861,3.883      /,
     I  (T(20,JJ),JJ=1,13) /0.127,0.257,0.391,0.533,0.687,0.860,
     J 1.064,1.325,1.725,2.0860,2.528,2.845,3.850      /
  DATA (T(21,JJ),JJ=1,13) /0.127,0.257,0.391,0.532,0.686,0.859,
     1 1.063,1.323,1.721,2.0796,2.518,2.831,3.819      /,
     2  (T(22,JJ),JJ=1,13) /0.127,0.256,0.390,0.532,0.686,0.858,
     3 1.061,1.321,1.717,2.0739,2.508,2.819,3.792      /,
     4  (T(23,JJ),JJ=1,13) /0.127,0.256,0.390,0.532,0.685,0.858,
     5 1.060,1.319,1.714,2.0687,2.500,2.807,3.767      /,
     6  (T(24,JJ),JJ=1,13) /0.127,0.256,0.390,0.531,0.685,0.857,
     7 1.059,1.318,1.711,2.0639,2.492,2.797,3.745      /,
     8  (T(25,JJ),JJ=1,13) /0.127,0.256,0.390,0.531,0.684,0.856,
     9 1.058,1.316,1.708,2.0595,2.485,2.787,3.725      /,
     A  (T(26,JJ),JJ=1,13) /0.127,0.256,0.390,0.531,0.684,0.856,
     B 1.058,1.315,1.706,2.0555,2.479,2.779,3.707      /,
     C  (T(27,JJ),JJ=1,13) /0.127,0.256,0.389,0.531,0.684,0.855,
     D 1.057,1.314,1.703,2.0518,2.473,2.771,3.690      /,
     E  (T(28,JJ),JJ=1,13) /0.127,0.256,0.389,0.530,0.683,0.855,
     F 1.056,1.313,1.701,2.0484,2.467,2.763,3.674      /,
     G  (T(29,JJ),JJ=1,13) /0.127,0.256,0.389,0.530,0.683,0.854,
```

```
     H      1.055,1.311,1.699,2.0452,2.462,2.756,3.659       /,
     I       (T(30,JJ),JJ=1,13) /0.127,0.256,0.389,0.530,0.683,0.854,
     J      1.055,1.310,1.697,2.0423,2.457,2.750,3.646       /
      DATA   (T(31,JJ),JJ=1,13) /0.126,0.255,0.388,0.529,0.681,0.851,
     1      1.050,1.303,1.684,2.0211,2.423,2.704,3.551       /,
     2       (T(32,JJ),JJ=1,13) /0.126,0.254,0.387,0.527,0.679,0.848,
     3      1.046,1.296,1.671,2.0003,2.390,2.660,3.460       /,
     4       (T(33,JJ),JJ=1,13) /0.126,0.254,0.386,0.526,0.677,0.845,
     5      1.041,1.289,1.658,1.9799,2.358,2.617,3.373       /,
     6       (T(34,JJ),JJ=1,13) /0.126,0.253,0.385,0.524,0.674,0.842,
     7      1.036,1.282,1.645,1.9600,2.326,2.576,3.291       /
C
C      T(II,JJ)    IS THE T-STATISTIC AT THE TABULATED DEGREES OF FREEDOM
C       (II)  AND AT THE TABULATED PROBABILITY LEVELS (JJ).
C        II=DEGREES OF FREEDOM, EXCEPT FOR
C        II=31 IS FOR 40 DEGREES
C        II=32 IS FOR 60
C        II=33 IS FOR 120
C        II=34 IS FOR INFINITY
C
C      JJ    PROBABILITY LEVEL       *      JJ    PROBABILITY LEVEL
C       1          0.10              *       8          0.80
C       2          0.20              *       9          0.90
C       3          0.30              *      10          0.95
C       4          0.40              *      11          0.98
C       5          0.50              *      12          0.99
C       6          0.60              *      13          0.999
C       7          0.70              *
C
C***********************************************************************
C      CALCULATE T STATISTICS
C
  220 WRITE (LIST,230)
  230 FORMAT(1H0,23HCALCULATED T STATISTICS /75H THE T STATISTICS CAN BE
     1 USED TO TEST THE NET REGRESSION COEFFICIENTS B(I). )
      DO 260 J=1,NOTERM
      DO 240 K=1,NODEP
      TT(J,K)=ABS(B(J,K)/DEVB(J,K))
  240 CONTINUE
      WRITE (LIST,250)(TT(J,K),K=1,NODEP)
  250 FORMAT(1H 9G14.6)
  260 CONTINUE
C
C***********************************************************************
      NDEG = NERROR
C
C***********************************************************************
C      SEARCH THE TABLE OF TABULATED DEGREES OF FREEDOM
C
      MAKENU=.FALSE.
      IF(NDEG-30)290,290,300
  290 II=NDEG
      GO TO 400
  300 IF(NDEG-40)310,320,330
  310 FINV=1.0/40.0
      FM1INV=1.0/30.0
```

```
      MAKENU=.TRUE.
  320 II=31
      GO TO 400
  330 IF(NDEG-60)340,350,360
  340 FINV=1.0/60.0
      FM1INV=1.0/40.0
      MAKENU=.TRUE.
  350 II=32
      GO TO 400
  360 IF(NDEG-120)370,380,390
  370 FINV=1.0/120.0
      FM1INV=1.0/60.0
      MAKENU=.TRUE.
  380 II=33
      GO TO 400
  390 II=34
      FINV=0.0
      FM1INV=1.0/120.0
      MAKENU=.TRUE.
C
C
  400 WRITE(LIST,410)
  410 FORMAT(104H UNDER NULL HYPOTHESIS THE INTERVAL (-T,T) WHERE T IS G
     XIVEN ABOVE, HAS APPROX PROBABILITY LISTED BELOW.      /42H  MINUS S
     XIGN INDICATES PROB EXCEEDS .999.    )
      IF(.NOT.MAKENU) GO TO 430
      FNDEG=NDEG
      DO  420   JJ=1,13
      T(35,JJ)=T(II,JJ)+((1.0/FNDEG - FINV)/(FM1INV-FINV))*(T(II-1,JJ)
     1 -T(II,JJ))
  420 CONTINUE
      II=35
  430 DO  560   J=1,NOTERM
      DO  540   K=1,NODEP
      DO 440   JJ=1,13
      IF(T(II,JJ)-TT(J,K))440,450,460
  440 CONTINUE
      PROB(J,K)=-0.999
      GO TO 540
  450 PROB(J,K)=PLEVEL(JJ)
      GO TC 540
  460 IF(JJ.LE.9) GO TO 470
      JJ1=JJ-2
      JJ2=JJ-1
      JJ3=JJ
      GO TO 490
  470 IF(JJ.LE.4)GO TO 480
      JJ1=JJ-1
      JJ2=JJ
      JJ3=JJ+1
      GO TO 490
  480 JJ1=JJ
      JJ2=JJ+1
      JJ3=JJ+2
C
C      PERFORM A THREE-POINT LAGRANGE INTERPOLATION
```

```
C
  490 X=ALOG(TT(J,K))
      X1=ALOG(T(II,JJ1))
      X2=ALOG(T(II,JJ2))
      X3=ALOG(T(II,JJ3))
      IF(TT(J,K).LE.1.0) GO TO 500
      Y1=ALOG(1.0-PLEVEL(JJ1))
      Y2=ALOG(1.0-PLEVEL(JJ2))
      Y3=ALOG(1.0-PLEVEL(JJ3))
      GO TO 510
  500 Y1=ALOG(PLEVEL(JJ1))
      Y2=ALOG(PLEVEL(JJ2))
      Y3=ALOG(PLEVEL(JJ3))
  510 PROB(J,K)= ((X-X2)*(X-X3)*Y1)/((X1-X2)*(X1-X3)) + ((X-X1)*(X-X3)
     1  *Y2)/((X2-X1)*(X2-X3))  + ((X-X1)*(X-X2)*Y3)/((X3-X1)*(X3-X2))
      IF(TT(J,K)-1.0) 520,520,530
  520 PROB(J,K)=EXP(PROB(J,K))
      GO TO 540
  530 PROB(J,K)=1.0-EXP(PROB(J,K))
  540 CONTINUE
C*********************************************************************
C     WRITE THE PROBABILITIES (1.0-ALPHA)
      WRITE(LIST,550) IDOUT(J),(PROB(J,K),K=1,NODEP)
  550 FORMAT(1H I3,9(8X,F6.3))
  560 CONTINUE
C
C*********************************************************************
C     LIST THE DESIRED VALUE OF PROBABILITY (PWANT)
C
  570 PERCEN=PWANT*100.0
      WRITE(LIST,580) PERCEN
  580 FORMAT(1H0,36HTHE DESIRED VALUE OF PROBABILITY IS ,F5.1, 8H PERCEN
     1T )
C
C     DELETE THE TERM WITH THE LOWEST COMPUTED PROBABILITY IF THAT
C     PROBABILITY IS LESS THAN THAT DESIRED (PWANT)
C
      IF(.NOT.DELETE) GO TO 660
      IOUT=0
  590 AMIN=PWANT
      DO 620    J=1,NOTERM
      IF(ABS(PROB(J,1))-PWANT)600,620,620
  600 IF(ABS(PROB(J,1))-AMIN)610,620,620
  610 AMIN=ABS(PROB(J,1))
      IOUT=J
  620 CONTINUE
      IF(IOUT) 660,660,630
  630 WRITE(LIST,650) IDOUT(IOUT)
  650 FORMAT(1H 10X,11HTHE TERM X(,I2,18H) IS BEING DELETED  )
      GO TO 670
C      ALL VARIABLES REMAINING HAVE BEEN CONCLUDED SIGNIFICANT
  660 RETURN1
  670 RETURN
      END
```

```
$IBFTC PRECIX
C
C      SUBROUTINE PREDIC
C
C      PURPOSE
C          1)READ INPUT LEVELS OF INDEPENDENT VARIABLES AND COMPUTE
C            A PREDICTED RESPONSE FROM THE ESTIMATED REGRESSION EQUATION.
C          2)COMPUTE VARIANCE AND STADARD DEVIATION OF THE PREDICTED
C            MEAN VALUE AND A SINGLE FURTHER OBSERVATION.
C
C      SUBROUTINES NEEDED
C            TRANS
C            LOC
C
C      REMARKS
C            VALUES FOR DEPENDENT VARIABLES ARE NOT NECCESARY FOR THE
C            PREDICTING OF VALUES. HOWEVER, A DUMMY VALUE MAY NEED TO
C            BE SUPPLIED IF A ZERO (BLANK) INPUT VALUE WILL CAUSE AN
C            IMPOSSIBLE OPERATION TO BE ATTEMPTED DURING THE
C            TRANSFORMATIONS.
C***********************************************************************
C
      SUBROUTINE PREDIC
      COMMON/BIG/SUMXX(1830),SUMXXI(1830),EIG(1830),SUMXY(60,9),
     X B(60,9),CORR(1830)
      COMMON/MED/        BO(9),             CON(99),            ERRMS(9),
     X   ICENT(13),IDOUT(60),NCON(200),
     X   NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                            RESMS(9),           SUMX(138),
     XSUMX2(69),         X(99),             ZEAN(69),           SUMY2(18)
       COMMON /FRMTS/ FMT(13),FMTTRI(14)
      COMMON/SMALL/      BYPASS,BZERO,DELETE, FIRST, IFCHI,  IFSSR,
     X       IFTT,       IFWT,       INPUT,      INPUT5,     INTER,
     X       ISTRAT,     JCOL,       KONNO,      LENGTH,     LIST,
     X       NERROR,     NODEP,      NOOB,                   NOTERM,
     X       NOVAR,      NPDEG,      NRES,       NTRANS,     NWHERE,
     X       P,          PREDCT,     REPS,       RWT,
     X       STORYI,      STORYC,     STORYX,     TOTWT,      WEIGHT,
     X       ERRFXD, ECONMY
      LOGICAL ECONMY
      LOGICAL      BYPASS,      BZERO,      DELETE,      IFCHI,
     XIFSSR,       IFTT,        IFWT,       REPS,        PREDCT,
     XSTORYC,      STORYX,      STORYI,     FIRST ,ERRFXD
      DOUBLE PRECISION WEIGHT,RWT,TOTWT
      COMMON/CNTRS/      I,          IBC,        IC,         ICOL,
     X       INEW,       INOCH,      IOLD,       IOUT,       IR,
     X       IRC,        IREP,       IS,         ITC,        J,
     X       K,          KBAR
      DIMENSION YCALC(9),     V(60),      VARM(9),      SEEM(9),
     X     VARP(9),     SEEP(9)
      EQUIVALENCE  (YCALC(1),SUMXX(1)),  (V(1),SUMXX(10)),
     X   (VARM(1),SUMXX(71)),   (SEEM(1),SUMXX(80)), (VARP(1),SUMXX(89))
     X   ,(SEEP(1),SUMXX(98))
      EQUIVALENCE (RNOOB,RWT)
C
C***********************************************************************
C
      IF(NOTERM.EQ.0) RETURN
      WRITE(6,3)
      READ(5,5) NPRED
C
      DO 500 KK=1,NPRED
```

```
C
  105 READ(5,FMT) (X(I),I=1,ICOL)
      WRITE(6,110)(X(I),I=1,ICOL)
  125 CALL TRANS
      DO 130 K=1,JCOL
      I=NTERM(K)
      X(K) = CON(I)
  130 CONTINUE
      WRITE(6,135) (X(I),I=1,NOTERM)
C
C     COMPUTE PREDICTED RESPONSE
  140 DO 150 K=1,NODEP
      YCALC(K) = BO(K)
      IF(.NOT.BZERO) YCALC(K)=0.0
      DO 150 J=1,NOTERM
      YCALC(K)= YCALC(K) + B(J,K)*X(J)
  150 CONTINUE
C
C     COMPUTE VARIANCE AND STANDARD DEVIATION OF REGRESSION LINE
C     AND VARIANCE AND STANDARD DEVIATION OF PREDICTED VALUE
C     AT THE POINT X0
C
      DO 250 K=1,NOTERM
      V(K)=0.0
      DO 250 J=1,NOTERM
      CALL LOC(J,K,IR)
      V(K)=V(K) + (X(J)-ZEAN(J))*SUMXXI(IR)
  250 CONTINUE
      XXT=0.0
      DO 275 K=1,NOTERM
      XXT = XXT + (X(K)-ZEAN(K))*V(K)
  275 CONTINUE
      XRNOOB = RNOOB
      IF(.NOT.BZERO) XRNOOB=0.0
      DO 300 K=1,NODEP
      VARM(K)= ERRMS(K)*(XRNOOB + XXT)
      SEEM(K)=SQRT(VARM(K))
      VARP(K)= ERRMS(K)+VARM(K)
      SEEP(K)=SQRT(VARP(K))
  300 CONTINUE
      WRITE(6,310)(YCALC(K),K=1,NODEP)
      WRITE(6,320)(VARM(K),K=1,NODEP)
      WRITE(6,320)(SEEM(K),K=1,NODEP)
      WRITE(6,320) (VARP(K),K=1,NODEP)
      WRITE(6,320) (SEEP(K),K=1,NODEP)
C
C
  500 CONTINUE
      RETURN
    3 FORMAT(54H1FOR EACH SET OF INDEP VARIABLES THERE IS COMPUTED...  /
     X  20H PREDICTED RESPONSE                                         /
     X 29H VARIANCE OF REGRESSION LINE                                 /
     X  34H STANDARD DEVIATION OF REGRESSION                           /
     X  29H VARIANCE OF PREDICTED VALUE                                /
     X  39H STANDARD DEVIATION OF PREDICTED VALUE      )
    5 FORMAT(I4)
  110 FORMAT(39HKINPUT DATA FOR THIS PREDICTED RESPONSE /(1H 9G14.6))
  135 FORMAT(56HK INDEPENDENT TERMS ACCORDING TO FINAL REGRESSION MODEL
     X  /(1H 9G14.6))
  310 FORMAT( 55HKPREDICTED RESPONSE FOR ABOVE INDEP VARIABLES
     X   /(1H 9G14.6))
  320 FORMAT(1H 9G14.6)
      END
```

```
$IBFTC CHISQX  LIST
C*********************************************************************
C
C     SUBRCUTINE CHISQ
C
C     PURPCSE
C
C         1)COMPUTE PREDICTED VALUE OF DEPENDENT VARIABLES AND RESIDUALS
C           AT INPUT DATA POINTS.
C         2) COMPUTE STANDARDIZED RESIDUALS.
C         3)COMPUTE SKEWNESS AND KURTOSIS OF SAMPLE DISTRIBUTION OF
C           RESIDUALS.  ALSO USE THE SAMPLE DISTRIBUTION TO COMPUTE
C           THE CHI-SQUARE STATISTIC.
C         4) PRINT HISTOGRAMS OF THE DISTRIBUTION OF RESIDUALS.
C
C    SUBROUTINES NEEDED
C      HIST
C
C*********************************************************************
C
      SUBROUTINE CHISQ
      COMMON/BIG/SUMXX(1830),SUMXXI(1830),EIG(1830),SUMXY(60,9),
     X  B(60,9),CORR(1830)
      COMMON/MED/         BO(9),             CON(99),          ERRMS(9),
     X   IDENT(13),IDOUT(60),NCON(200),
     X    NTERM(60), NTRAN(100), NXCOD(100), POOLED(9),
     XREPVAR(9),                             RESMS(9),         SUMX(138),
     XSUMX2(69),          X(99),             ZEAN(69),         SUMY2(18)
      COMMON/SMALL/       BYPASS,BZERO,DELETE, FIRST, IFCHI,   IFSSR,
     X      IFTT,         IFWT,        INPUT,       INPUT5,      INTER,
     X      ISTRAT,       JCOL,        KONNO,       LENGTH,      LIST,
     X      NERROR,       NODEP,       NOOB,                     NOTERM,
     X      NOVAR,        NPDEG,       NRES,        NTRANS,      NWHERE,
     X      P,            PREDCT,      REPS,        RWT,
     X      STORYI,        STORYC,      STORYX,      TOTWT,       WEIGHT,
     X      ERRFXD, ECONMY
      LOGICAL ECONMY
      DOUBLE PRECISION RWT,TOTWT,WEIGHT
      LOGICAL      BYPASS,      BZERO,       DELETE,      IFCHI,
     XIFSSR,       IFTT,        IFWT,        REPS,        PREDCT,
     XSTORYC,      STORYX,      STORYI,      FIRST ,ERRFXD
      COMMON/CNTRS/       I,           IBC,         IC,          ICOL,
     X      INEW,         INOCH,       IOLD,        IOUT,        IR,
     X      IRC,          IREP,        IS,          ITC,         J,
     X      K,            KBAR
C
C*********************************************************************
      INTEGER      CELLS,      PLUS1
      DIMENSION           BOUND(45),         CELLBD(21),       CHI(9),
     X OBS(20,9),         RCT(212),          RELKUR(9),        RELSKW(9),
     X STDERR(9),         VAR(9),            YCALC(9),         YDIFR(9),
```

```
     X Z(9)
      EQUIVALENCE (CELLBD,SUMXX(1)),       (CHI,SUMXX(22)),
     X            (OBS,SUMXX(31)),         (RCT,SUMXX(221)),
     X            (RELKUR,SUMXX(434)),     (RELSKW,SUMXX(443)),
     X            (STDERR,SUMXX(452)),     (VAR,RESMS),
     X            (YCALC,SUMXX(461)),      (YDIFR,SUMXX(470)),
     X            (Z,SUMXX(479))
     0DATA  BOUND/.67448907         ,.43072720         ,.96741604
     1,.31863932 ,.67448907         ,1.15033859        ,.25334708
     2,.52439979 ,.84162001         ,1.28153777        ,.21042836
     3,.43072721 ,.67448922         ,.96741746         ,1.38298075
     4,.18001235 ,.36610621         ,.56594856         ,.79163735
     5,1.06756653,1.46521688        ,.15731067         ,.31863932
     6,.48877614 ,.67448930         ,.88714436         ,1.15034184
     7,1.53411831,.13971028         ,.28221612         ,.43072722
     8,.58945544 ,.76470731         ,.96741836         ,1.22062834
     9,1.59323335,.12566134         ,.25334701         ,.38532026
     A,.52440000 ,.67448801         ,.84161868         ,1.03642921
     B,1.28154233,1.64490172  /
C
      JCOL=NOTERM+ NODEP
      NUVAR=NOTERM+1
      BYPASS= .FALSE.
      KOUNT= 0
C
C*************************************************************************
C     DETERMINE IF SAMPLE SIZE IS LARGE ENOUGH TO PERMIT CHI-SQUARE
C     CALCULATION. IF SO, DETERMINE NUMBER OF CELLS AND CELL BOUNDARIES
      IF(NERROR-30) 110,120,120
  110 BYPASS=.TRUE.
      GO TO 125
  120 CELLS=NOOB/5
      CELLS=MIN0(CELLS,20)
      I= MOD(CELLS,2)
      IF(I.NE.0) CELLS=CELLS + I
      FCELLS= FLOAT(CELLS)
      PLUS1= CELLS + 1
      MINUS1 = CELLS -1
      NDEGCH = CELLS-3
      IR= CELLS/2-1
      IC=IR*(IR-1)/2
      IS=IR+2
      DO 122 J=1,IR
      IC=IC+1
      IBC=IS-J
      IRC=IS+J
      CELLBD(IBC)=-BOUND(IC)
      CELLBD(IRC)= BOUND(IC)
  122 CONTINUE
      CELLBD(1)=-1.0E+37
      CELLBD(PLUS1) =1.0E37
      CELLBD(IS  )=0.0
      DO 124 K=1,NODEP
      CHI(K)=0.0
      DO 124 I=1,CELLS
      OBS(I,K)=0.0
```

```
  124 CONTINUE
C
C**********************************************************************
C      INITIALIZE SKEWNESS AND KURTOSIS ARRAYS.  COMPUTE STANDARD ERROR
C      OF ESTIMATE.
  125 DO 130 K=1,NODEP
      RELKUR(K)=0.0
      RELSKW(K)=0.0
      STDERR(K)= SQRT(ERRMS(K))
  130 CONTINUE
      WRITE(LIST,135)
  135 FORMAT( 51H FOR EACH DEPENDENT TERM AND OBSERVATION IS PRINTED
     X     /31H OBSERVED RESPONSE (Y OBSERVED)
     X     /29H CALCULATED RESPONSE (Y CALC)
     X     /28H RESIDUAL (YOBS- YCALC=YDIF)
     X     /28H STANDARDIZED RESIDUAL ( Z ) )
C
C**********************************************************************
      DO 430 J=1,NOOB
      READ(INTER) (X(I),I=1,69  ),WEIGHT
      DO 142 I=1,NOTERM
      K= IDOUT(I)
      X(I)= X(K)
  142 CONTINUE
      KBAR=NWHERE
      DO 143 I=1,NODEP
      IC= NOTERM+ I
      KBAR=KBAR+1
      X(IC)= X(KBAR)
  143 CONTINUE
C
C*************************************************
      DO 160 K= 1,NODEP
      YCALC(K)= BO(K)
      IF(.NOT.BZERO) YCALC(K)= 0.0
      KBAR= K+NOTERM
      DO 150 I=1,NOTERM
      YCALC(K) = YCALC(K) + B(I,K)*X(I)
  150 CONTINUE
      ACTDEV= X(KBAR)- YCALC(K)
      YDIFR(K)= ACTDEV
      Z(K)=ACTDEV/STDERR(K)
      ACTDE3=ACTDEV**3
      RELSKW(K)= RELSKW(K)+ACTDE3
      RELKUR(K) = RELKUR(K)+ACTDE3*ACTDEV
  160 CONTINUE
      WRITE(LIST,180) (X(K),K=NUVAR,JCOL)
      WRITE(LIST,190) (YCALC(K),K=1,NODEP)
      WRITE(LIST,200) (YDIFR(K),K=1,NODEP)
      WRITE(LIST,210) (Z(K),K=1,NODEP)
  180 FORMAT(12HKY OBSERVED  ,9G13.4)
  190 FORMAT(12H Y CALC      ,9G13.4)
  200 FORMAT(12H Y DIF       ,9G13.4)
  210 FORMAT(12H STUDENTIZED ,9G13.4)
      IF(BYPASS) GO TO 410
C
```

```
C*************************************************
      DO 250 K=1,NODEP
      DO 230 I=1,PLUS1
      IF(Z(K)-CELLBD(I)) 220,220,230
  220 OBS(I-1,K)=OBS(I-1,K)+ 1.0
      GO TO 250
  230 CONTINUE
  250 CONTINUE
C
  410 KOUNT = KOUNT +1
      IF(KOUNT.LT.10) GO TO 430
      WRITE(LIST,270) IDENT
  270 FORMAT(1H113A6,A2)
      KOUNT=0
  430 CONTINUE
C
C**********************************************************************
C     PRINT SKEWNESS AND KURTOSIS
      DO 440 K=1,NODEP
      RELSKW(K)=RELSKW(K)**2/(FLOAT(NOOB)**2*ERRMS(K)**3)
      RELKUR(K)=RELKUR(K)/(FLOAT(NOOB)*ERRMS(K)**2)
  440 CONTINUE
      WRITE(LIST,450) IDENT,(RELSKW(K),K=1,NODEP)
  450 FORMAT( 1H1 13A6//10X,30HSKEWNESS (SHOULD BE NEAR ZERO)      //
     X     12X,9F12.4)
      WRITE(LIST,460) (RELKUR(K),K=1,NODEP)
  460 FORMAT(10X,31HKURTOSIS (SHOULD BE NEAR THREE)     //12X,9F12.4)
      IF(.NOT.BYPASS) GO TO 480
      WRITE(LIST,470)
  470 FORMAT(74HKCHI-SQUARE IS NOT COMPUTED FOR LESS THAN 30 DEGREES OF
     XFREEDOM FOR ERROR. )
      RETURN
C
C**********************************************************************
C     COMPUTE CHI-SQUARED AND PRINT HISTOGRAMS OF RESIDUALS
  480 DO 580 K=1,NODEP
      DO 570 I=1,CELLS
      CHI(K)= CHI(K) +OBS(I,K)**2
  570 CONTINUE
      CHI(K)=FCELLS*CHI(K)/FLOAT(NOOB)-FLOAT(NOOB)
  580 CONTINUE
      WRITE(LIST,590) NDEGCH,(CHI(K),K=1,NODEP)
  590 FORMAT( 55HKTHE CHI-SQUARED VALUES ARE LISTED BELOW.  COMPARE WITH
     X   I10,20H  DEGREES OF FREEDOM /  1H 9G14.6)
C
      RELFRQ = TOTWT/FCELLS
      DO 650 K=1,NODEP
      WRITE(LIST,620) K,K,RELFRQ
  620 FORMAT(1H1,I5,18H   DEPENDENT TERM ,I5, 59H   IF THE DISTRIBUTION
     XWERE NORMAL EACH CELL WOULD CONTAIN  F6.2, 8H  COUNTS )
      DO 640  I=1,CELLS
      RCT(I)=OBS(I,K)
  640 CONTINUE
      CALL HIST(K,RCT,CELLS)
  650 CONTINUE
      RETURN
      END
```

```
$IBFTC RECTXX

      SUBROUTINE RECT(IROW,JJCOL,IMAX,JMAX,A,FMT)
      DIMENSION  A(IMAX,JMAX),FMT(14),XOUT(8)
      DATA J8/8/
      LOGICAL OUT
      OUT =.FALSE.
      JTIMES=0
      JCOL=JJCOL
    5 JNXT=JCOL-J8
      IF(JNXT) 10,20,30
   10 JP=JCOL
      GO TO 40
   20 JP=J8
      GO TO 40
   30 JCOL=JNXT
      JP=J8
      GO TO 50
   40 OUT=.TRUE.
   50 DO 100 I=1,IROW
      DO 60 J=1,JP
      JJ=JTIMES +J
   60 XOUT(J) = A(I,JJ)
      WRITE (6,FMT) I,(XOUT(K),K=1,JP)
  100 CONTINUE
      IF(OUT) RETURN
      WRITE(6,110)
  110 FORMAT(1H /1H )
      JTIMES=JTIMES +JP
      GO TO 5
      END
```

```
$IBFTC LOCXXX

      SUBROUTINE LOC(I,J,IR)
      IX= I
      JX= J
   20 IF(IX-JX) 22,24,24
   22 IRX= IX + (JX*JX-JX)/2
      GO TO 36
   24 IRX= JX + (IX*IX - IX)/2
   36 IR= IRX
      RETURN
      END
```

```
$IBFTC BORCXX

C
C      SUBROUTINE BORD
C
C      PURPOSE
C            TO COMPLETE THE INVERSION OF A SYMMETRIC POSITIVE DEFINITE
C            MATRIX A OF ORDER N GIVEN THAT THE UPPER LEFT SUB-
C            MATRIX OF ORDER N-1 HAS ALREADY BEEN INVERTED.
C
C      SUBROUTINES NEEDED
C            LOC
C
C      REMARKS
C            ONLY THE UPPER TRIANGULAR PART OF A IS STORED AS A
C            VECTOR IN THE ORDER A(1,1),A(1,2),A(2,2),A(1,3),...ETC
       SUBROUTINE BORD(IORDER,A)
C
C
       DIMENSION  BETA(60),A(1)
C
C
       ALPHA= 0.0
       NM1= IORDER-1
       IF(NM1) 100,100,200
   100 A(1) = 1.0/A(1)
       GO TO 600
   200 M=NM1*(NM1+1)/2
       LEN = M + IORDER
C
       DO 400 I=1,NM1
       BETA(I)= 0.0
       MI= M+I
       DO 350 J=1,NM1
       CALL LOC(I,J,II)
       MJ= M+J
       BETA(I)= BETA(I)-A(II)*A(MJ)
   350 CONTINUE
       ALPHA= ALPHA + A(MI)*BETA(I)
   400 CONTINUE
C
C
        ALPHA = ALPHA + A(LEN)
       RALPHA= 1.0/ALPHA
       A(LEN) = RALPHA
C
       DO 500 I=1,NM1
       DO 500 J=1,I
       CALL LOC(I,J,II)
       A(II)= A(II) + BETA(I)*BETA(J)*RALPHA
   500 CONTINUE
C
       DO 550 J=1,NM1
       MJ= M+J
       A(MJ)= BETA(J)*RALPHA
   550 CONTINUE
C
C
   600 CONTINUE
       RETURN
       END
```

APPENDIX B

BORROWED ROUTINES

Some of the routines used in the program were taken from the literature. Both INVXTX and TRIANG are by Webb and Galley (ref. 9), and EIGEN and HIST are from the IBM programmer's manual (ref. 10).

Listing of INVXTX and TRIANG are given here, as follows:

```
$IBFTC INVXXX

      SUBROUTINE INVXTX(A, NN, D, FACT)
C
C     ASSUMES THE MATRIX A IS SYMMETRIC AND POSITIVE DEFINITE, AND ONLY
C     THE  UPPER TRIANGLE IS STORED AS A ONE-DIMENSIONAL ARRAY IN THE
C     ORDER A(1,1), A(1,2), A(2,2), A(1,3), A(2,3), A(3,3), ..., A(N,N).
C     NN IS THE ORDER N OF THE INPUT MATRIX A.
C     D  IS (ON EXIT) THE DETERMINANT OF A, DIVIDED BY FACTOR**NN.
C
      DIMENSION A(1)
      D =  1.0D0
      N = NN
      ITR1 = 0
      FACTOR =  FACT
      DO 145 K=1,N
C
      ITR1 = ITR1+K-1
      KP1 = K+1
      KM1 = K-1
      KK = ITR1+K
C     CONTINUED PRODUCT OF PIVOTS
      D = D*A(KK)/FACTOR
      PV =  1.0D0/A(KK)
C
      ITR2 = 0
      IF (K-1) 150,80,50
C
C     REDUCE TOP PART OF TRIANGLE, LEFT OF PIVOTAL COLUMN
   50 DO 60 J=1,KM1
      ITR2 = ITR2+J-1
      KJ = ITR1+J
      F = A(KJ)*PV
      DO 60 I=1,J
      IJ = ITR2+I
      IK = ITR1 + I
   60 A(IJ) = A(IJ) + A(IK)*F
C
      IF (K-N) 70,120,150
C
C     REDUCE REST OF TRIANGLE, RIGHT OF PIVOTAL COLUMN
   70 ITR2 = ITR1
   80 DO 110 J=KP1,N
      ITR3 = ITR1
      ITR2 = ITR2+J-1
      KJ = ITR2+K
      F = A(KJ)*PV
      DO 100 I=1,J
      IF (I-K) 90,100,95
   90 IJ = ITR2+I
      IK = ITR1 + I
      A(IJ) = A(IJ) - A(IK)*F
      GO TO 100
   95 IJ = ITR2 + I
      ITR3 = ITR3 + I - 1
      IK = ITR3 + K
      A(IJ) = A(IJ) - A(IK)*F
  100 CONTINUE
  110 CONTINUE
C
C     DIVIDE PIVOTAL ROW-COLUMN BY PIVOT, INCLUDING APPROPRIATE SIGNS
  120 ITR2 = ITR1
```

```
      DO 140 I=1,N
      IF (I-K) 125,130,135
  125 IK = ITR1+I
      A(IK) = -A(IK)*PV
      GO TO 140
C     (REPLACE PIVOT BY RECIPROCAL)
  130 A(KK) = PV
      GO TO 140
  135 ITR2 = ITR2+I-1
      KI = ITR2+K
      A(KI) =  A(KI)*PV
  140 CONTINUE
C
  145 CONTINUE
C
  150 RETURN
      END
```

```
$IBFTC TRIANX
      SUBROUTINE TRIANG(A,NN,NKOL,FORMAT)
      DIMENSION FORMAT(1)
      DIMENSION A(1)
    1 FORMAT (1H1)
    3 FORMAT(1H /1H /1H   )
      N = NN
      NCOL = NKOL
      KLUMPS = N/NCOL
C
      KEEPTR = 0
      K1 = 1
      K2 = NCOL - 1
      K3 = NCOL
      IF (KLUMPS .EQ. 0)  GO TO 120
C
      DO 90 KLUMP=1,KLUMPS
      ITR1 = KEEPTR
      I = -1
      ILO = (KLUMP-1)*NCOL + ITR1 + 1
      DO 30 K=K1,K2
      I = I + 1
      ITR1 = ITR1 + K - 1
      ILO  = ILO  + K - 1
      IHI = ILO + I
   30 WRITE (6,FORMAT) K, (A(J), J=ILO,IHI)
      KEEPTR = ITR1 + K2
      DO 60 K=K3,N
      ITR1 = ITR1 + K - 1
      ILO  = ILO  + K - 1
      IHI = ILO + NCOL - 1
   60 WRITE (6,FORMAT) K, (A(J), J=ILO,IHI)
      K1 = K1 + NCOL
      K2 = K2 + NCOL
      K3 = K3 + NCOL
   90 WRITE(6,3)
C
  120 ITR1 = KEEPTR
      IF (K1 .GT. N)  GO TO 180
      I = -1
      ILO = KLUMPS*NCOL + ITR1 + 1
      DO 150 K=K1,N
      I = I + 1
      ITR1 = ITR1 + K - 1
      ILO  = ILO  + K - 1
      IHI = ILO + I
  150 WRITE (6,FORMAT) K, (A(J), J=ILO,IHI)
C
  180 RETURN
      END
```

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